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INTRODUCTION

In the course of running a business, decisions are made in the presence of risk. A decision maker can confront one of two types of risk. Some risks are related to the underlying nature of the business and deal with such matters as the uncertainty of future sales or the cost of inputs. These risks are called business risks. Most businesses are accustomed to accepting business risks. Indeed, the acceptance of business risks and its potential rewards are the foundations of capitalism. Another class of risks deals with uncertainties such as interest rates, exchange rates, stock prices, and commodity prices. These are called financial risks.

Financial risks are a different matter. The paralyzing uncertainty of volatile interest rates can cripple the ability of a firm to acquire financing at a reasonable cost, which enable it to provide its products and services. Firms that operate in foreign markets can have excellent sales performance offset if their own currency is strong. Companies that use raw materials can find it difficult to obtain their basic inputs at a price that will permit profitability. Managers of stock portfolios deal on a day-to-day basis with wildly unpredictable and sometimes seemingly irrational financial markets.

Although our financial system is replete with risk, it also provides a means of dealing with risk in the form of derivatives. Derivatives are financial instruments whose returns are derived from those of other financial instruments. That is, their performance depends on how other financial instruments perform. Derivatives serve a valuable purpose in providing a means of managing financial risk. By using derivatives, companies and individuals can transfer, for a price, any undesired risk to other parties who either have risks that offset or want to assume that risk.

Although derivatives have been around in some form for centuries, their growth has accelerated rapidly during the last several decades. They are now widely used by corporations, financial institutions, professional investors, and individuals. Certain types of derivatives are traded actively in public markets, similar to the stock exchanges with which you are probably already somewhat familiar. The vast majority of derivatives, however, are created in private transactions in over-the-counter markets. Just as a corporation may buy a tract of land for the purpose of ultimately putting up a factory, so may it also engage in a derivatives transaction. In neither case is the existence or amount of the transaction easy for outsiders to determine. Nonetheless, we have fairly accurate data on the amount of derivatives activity in public markets and reasonably accurate data, based on surveys, on the amount of derivatives activity in private markets. We shall explore the public market data in later chapters. If you need to be convinced that derivatives are worth studying, consider this: *The Bank for International Settlements of Basel, Switzerland, estimated that at the end of 2005, over-the-counter derivatives contracts outstanding worldwide covered underlying assets of over $285 trillion. In comparison, gross domestic product in the United States at the end of 2005 was about $13 trillion.* As we shall see later, measuring the derivatives market this way can give a false impression of the size of the market. Nonetheless, the market value of these contracts totals about $9.1 trillion, making the derivatives market a sizable force in the global economy.

This book is an introductory treatment of derivatives. Derivatives can be based on real assets, which are physical assets and include agricultural commodities, metals, and sources of energy. Although a few of these will come up from time to time in this book, our focus will be directed towards derivatives on financial assets, which are stocks, bonds/loans, and currencies. In this book you will learn about the characteristics of the institutions and markets where these instruments trade, the manner in which derivative prices are determined, and the strategies in which they are used. Toward the end of the book, we will cover the way in which derivatives are used to manage the risk of a company.

This chapter welcomes you to the world of derivatives and provides an introduction to or a review of some financial concepts that you will need in order to understand derivatives. Let us begin by exploring the derivatives markets more closely and defining what we mean by these types of instruments.

DERIVATIVE MARKETS AND INSTRUMENTS

An asset is an item of ownership having positive monetary value. A liability is an item of ownership having negative monetary value. The term "instrument" is used to describe either assets or liabilities. Instrument is the more general term, vague enough to encompass the underlying asset or liability of derivative contracts. A contract is an enforceable legal agreement. A security is a tradeable instrument representing a claim on a group of assets.

In the markets for assets, purchases and sales require that the underlying asset be delivered either immediately or shortly thereafter. Payment usually is made immediately, although credit arrangements are sometimes used. Because of these characteristics, we refer to these markets as cash markets or spot markets. The sale is made, the payment is remitted, and the good or security is delivered. In other situations, the good or security is to be delivered at a later date. Still other types of arrangements allow the buyer or seller to choose whether or not to go through with the sale. These types of arrangements are conducted in derivative markets.

In contrast to the market for assets, derivative markets are markets for contractual instruments whose performance is determined by the way in which another instrument or asset performs. Notice that we referred to derivatives as contracts. Like all contracts, they are agreements between two parties—a buyer and a seller—in which each party does something for the other. These contracts have a price, and buyers try to buy as cheaply as possible while sellers try to sell as dearly as possible. This section briefly introduces the various types of derivative contracts: options, forward contracts, futures contracts, and swaps and related derivatives.

Options

An option is a contract between two parties—a buyer and a seller—that gives the buyer the right, but not the obligation, to purchase or sell something at a later date at a price agreed upon today.

The option buyer pays the seller a sum of money called the price or premium. The option seller stands ready to sell or buy according to the contract terms if and when the buyer so desires. An option to buy something is referred to as a call; an option to sell something is called a put. Although options trade in organized markets, a large amount of option trading is conducted privately between two parties who find that contracting with each other may be preferable to a public transaction on the exchange. This type of market, called an over-the-counter market, was actually the first type of options market. The creation of an organized options exchange in 1973 reduced the interest in over-the-counter option markets; however, the over-the-counter market has been revived and is now very large and widely used, mostly by corporations and financial institutions.

Most of the options that we shall focus on in this textbook are options that trade on organized options exchanges, but the principles of pricing and using options are very much the same, regardless of where the option trades. Many of the options of most interest to us are for the purchase or sale of financial assets, such as stocks or bonds. However, there are also options on futures contracts, metals, and foreign currencies. Many other types of financial arrangements such as lines of credit, loan guarantees, and insurance, are forms of options. Moreover, stock itself is equivalent to an option on the firm's assets.

Forward Contracts

A forward contract is a contract between two parties—a buyer and a seller—to purchase or sell something at a later date at a price agreed upon today. A forward contract sounds a lot like an option, but an option carries the right, not the obligation, to go through with the transaction. If the price of the underlying good changes, the option holder may decide to forgo buying or selling at the fixed price. On the other hand, the two parties in a forward contract incur the obligation to ultimately buy and sell the good.

Although forward markets have existed for a long time, they are somewhat less familiar. Unlike options markets, they have no physical facilities for trading; there is no building or formal corporate body organized as the market. They trade strictly in an over-the-counter market consisting of direct communications among major financial institutions.

Forward markets for foreign exchange have existed for many years. With the rapid growth of derivative markets, we have seen an explosion of growth in forward markets for other instruments. It is now just as easy to enter into forward contracts for a stock index or oil as it was formerly to trade foreign currencies. Forward contracts are also extremely useful in that they facilitate the understanding of futures contracts.

Futures Contracts

A futures contract is also a contract between two parties—a buyer and a seller—to buy or sell something at a future date at a price agreed upon today. The contract trades on a futures exchange and is subject to a daily settlement procedure. Futures contracts evolved out of forward contracts and possess many of the same characteristics. In essence, they are like liquid forward contracts. Unlike forward contracts, however, futures contracts trade on organized exchanges, called futures markets. For example, the buyer of a futures contract, who has the obligation to buy the good at the later date, can sell the contract in the futures market, which relieves her of the obligation to purchase the good. Likewise, the seller of a futures contract, who is obligated to sell the good at the later date, can buy the contract back in the futures market, relieving him of the obligation to sell the good.

Futures contracts also differ from forward contracts in that they are subject to a daily settlement procedure. In the daily settlement, investors who incur losses pay the losses every day to investors who make profits. Futures prices fluctuate from day to day, and contract buyers and sellers attempt both to profit from these price changes and to lower the risk of transacting in the underlying goods. We shall learn more about this in Chapter 8.

Options on futures, sometimes called *commodity options* or *futures options,* are an important synthesis of futures and options markets. An option on a futures contract gives the buyer the right to buy or sell a futures contract at a later date at a price agreed upon today. Options on futures trade on futures exchanges, and are a rare case where the derivative contract and the instrument on which it is derived trade side by side in an open market. Although options on futures are quite similar to options on spot assets, there are a few important differences, which we shall explore later in this book.

Swaps and Other Derivatives

Although options, forwards, and futures compose the set of basic instruments in derivative markets, there are many more combinations and variations. One of the most popular is called a swap. A swap is a contract in which two parties agree to exchange cash flows. For example, one party is currently receiving cash from one investment but would prefer another type of investment in which the cash flows are different. The party contacts a swap dealer, a firm operating in the over-the-counter market, who takes the opposite side of the transaction. The firm and the dealer, in effect, swap cash flow streams. Depending on what later happens to prices or interest rates, one party might gain at the expense of the other. In another type of arrangement, a firm might elect to tie the payments it makes on the swap contract to the price of a commodity, called a commodity swap. Alternatively, a firm might buy an option to enter into a swap, called a swaption. As we shall show later, swaps can be viewed as a combination of forward contracts, and swaptions are special types of options.

Interest rate swaps make up more than half of the $285 trillion notional principal over-the-counter derivatives market. But interest rate swaps are only one of many types of contracts that combine elements of forwards, futures, and options. For example, a firm that borrows money at a floating rate is susceptible to rising interest rates. It can reduce that risk, however, by buying a cap, which is essentially an option that pays off whenever interest rates rise above a threshold. Another firm may choose to purchase an option whose performance depends not on how one asset performs but rather on the better or worse performing of two, or even more than two, assets; this is called an alternative option.

Some of these types of contracts are referred to as hybrids because they combine the elements of several other types of contracts. All of them are indications of the ingenuity of participants in today's financial markets, who are constantly creating new and useful products to meet the diverse needs of investors. This process of creating new financial products is sometimes referred to as financial engineering. These hybrid instruments represent the effects of progress in our financial system. They are examples of change and innovation that have led to improved opportunities for risk management. Swaps, caps, and many other hybrid instruments are covered in Chapters 12, 13, and 14.

THE UNDERLYING ASSET

All derivatives are based on the random performance of something. That is why the word "derivative" is appropriate. The derivative *derives* its value from the performance of something else. That "something else" is often referred to as the *underlying asset.* The term underlying asset, however, is somewhat confusing and misleading. For instance, the underlying asset might be a stock, bond, currency, or commodity, all of which are assets. However, the underlying "asset" might also be some other random element such as the weather, which is not an asset. It might even be another derivative, such as a futures contract or an option. Hence, to avoid saying that a derivative is on an "underlying something," we corrupt the word "underlying," which is an adjective, and treat it as a noun. Thus, we say that the derivative is "on an underlying." This incorrect use of the word "underlying" serves a good purpose, however, because it enables us to avoid using the word "asset."

IMPORTANT CONCEPTS IN FINANCIAL AND DERIVATIVE MARKETS

Before undertaking any further study of derivative markets, let us review some introductory concepts pertaining to investment opportunities and investors. Many of these ideas may already be familiar and are usually applied in the context of trading in stocks and bonds. These concepts also apply with slight modifications to trading in derivatives.

Risk Preference

Suppose you were faced with two equally likely outcomes. If the first outcome occurs, you receive \$5. If the second outcome occurs, you receive \$2. From elementary statistics, you know that the expected outcome is $\$5(0.5) + \$2(0.5) = \$3.50$, which is the amount you would expect to receive on average after playing the game many times. How much would you be willing to pay to take this risk? If you say \$3.50, you are not recognizing the risk inherent in the situation. You are simply saying that a fair trade would be for you to give up \$3.50 for the chance to make \$3.50 on average. You would be described as risk neutral, meaning that you are indifferent to the risk. Most individuals, however, would not find this a fair trade. They recognize that the \$3.50 you pay is given up for certain, while the \$3.50 you expect to receive is earned only on average. In fact, if you play twice, lose \$1.50 once and then gain it back, you will feel worse than if you had not played.

Thus, we say that most individuals are characterized by risk aversion. They would pay less than $3.50 to take this risk. How much less depends on how risk averse they are. People differ in their degrees of risk aversion. But let us say you would pay $3.25. Then the difference between $3.50 and $3.25 is considered the risk premium. This is the additional return you expect to earn on average to justify taking the risk.

Although most individuals are indeed risk averse, it may surprise you to find that in the world of derivative markets, we can actually pretend that most people are risk neutral. No, we are not making some heroic but unrealistic assumption. It turns out that we obtain the same results in a world of risk aversion as we do in a world of risk neutrality. Although this is a useful point in understanding derivative markets, we shall not explore it in much depth at the level of this book. Yet without realizing it, you will probably grow to accept and understand derivative models and the subtle implication of risk neutrality.

Short Selling

If you have already taken an investments course, you probably have been exposed to the idea of short selling. Short selling is an important transaction related to making a market in derivatives. Therefore, the costs related to short selling have a direct impact on derivative pricing. Nonetheless, the concept is not too straightforward, and a little review will be beneficial.

A typical transaction in the stock market involves one party buying stock from another party. It is possible, however, that the party selling the stock does not actually own the stock. That party could borrow the stock from a broker. That person is said to be selling short or, sometimes, *shorting*. She is doing so in the anticipation of the price falling, at which time the short seller would then buy back the stock at a lower price, capturing a profit and repaying the shares to the broker. You may have heard the expression, "Don't sell yourself short," which simply means not to view yourself as being less talented or less correct than someone else. Similarly, a short seller views the stock as being worth less than the market price.

Establishing a short position creates a liability. The short seller is obligated to someday buy back the stock and return it to the broker. Unlike an ordinary loan in which a borrower knows exactly how much he or she must pay back the lender, the short seller does not know how much he or she will have to pay to buy back the shares. This makes it a rather risky type of borrowing. Indeed short selling is a very daring investment strategy.

Short selling, however, can be quite beneficial in that the risk of short positions can be useful in offsetting the risk of long positions. Alternatively, taking a short position in a derivative may be more efficient. Short selling of stocks can be quite complex and expensive relative to buying stocks, whereas taking a short position in a derivative is as simple as buying derivatives. Short selling of stocks requires finding someone willing to lend you the stock. At times, security lending can be expensive. Thus, it is common to find an investor holding a stock and protecting it by selling a derivative.

We should note that anyone who has an obligation to purchase something at a later date has the equivalent of a short sale. It is not necessary to have borrowed stock from a broker. In either case an increase in the price will be harmful.

The terminology of short selling can be confusing. In the context of financial securities, short selling, shorting, or going short are synonymous. In the context of derivative contracts, shorting or going short are synonymous. We do not refer to selling derivative contracts as short selling because the underlying security is not borrowed.

Repurchase Agreements

A repurchase agreement (known as repos) is a legal contract between a seller and a buyer; the seller agrees to sell currently a specified asset to the buyer—as well as buy it back (usually) at a specified time in the future at an agreed future price. The seller is effectively borrowing money from the buyer at an implied interest rate. Typically, repos involve low-risk securities, such as U.S. Treasury bills. Repos are useful because they provide a great deal of flexibility to both the borrower and lender.

Derivatives traders often need to be able to borrow and lend money in the most cost-effective manner possible. Repos are often a very low-cost way of borrowing money, particularly if the firm holds government securities. Repos are a way to earn interest on short-term funds with minimal risk (for buyers), as well as a way to borrow for short-term needs at a relatively low cost (for sellers).

As we will see in subsequent chapters, derivative market participants must often rely on the ability to borrow and lend money on a short-term basis. Many derivative valuation models are based on the assumption that the price-setting trader, often a dealer, has access to money or can lend money at the risk-free rate. The repo rate is an approximation of the dealer's marginal cost of funds, and hence is a good approximation of the dealer's cost of borrowing and lending. Also, due to the strong collateral used in the repo market, the repo rate is roughly analogous to the government rate.

Return and Risk

Return is the numerical measure of investment performance. There are two main measures of return, dollar return and percentage return. Dollar return measures investment performance as total dollar profit or loss. For example, the dollar return for stocks is the dollar profit from the change in stock price plus any cash dividends paid. It represents the absolute performance. Percentage return measures investment performance per dollar invested. It represents the percentage increase in the investor's wealth that results from making the investment. In the case of stocks, the return is the percentage change in price plus the dividend yield. The concept of return also applies to options, but, as we shall see later, the definition of the return on a futures or forward contract is somewhat unclear.

One fundamental characteristic of investors is their desire to increase in wealth. This translates into obtaining the highest return possible—but higher returns are accompanied by greater risk. Risk is the uncertainty of future returns. As we noted earlier, investors generally dislike risk, and they demonstrate this characteristic by avoiding risky situations when riskless ones that offer equivalent expected returns exist; however, they cannot always avoid uncertainty. Fortunately, the competitive nature of financial and derivative markets enables investors to identify investments by their degrees of risk.

For example, the stock of a company that specializes in drilling wildcat oil wells will, all other things being equal, sell for less than the stock of a company that supplies health care.[1] The stock price is lower due to the drilling company's more uncertain line of business. Risk, of course, runs the spectrum from minimal risk to high risk. The prices of securities will reflect the differences in the companies' risk levels. The additional return one expects to earn from assuming risk is the risk premium, which we mentioned earlier.

What other factors influence a company's stock price and expected return? Consider a hypothetical company with no risk. Will people be willing to invest money in this company if they expect no return? Certainly not. They will require a minimum return, one sufficient to compensate them for giving up the opportunity to spend their money today. This return is called the risk-free rate and is the investment's opportunity cost.[2]

The return investors expect is composed of the risk-free rate and a risk premium. This relationship is illustrated in Figure 1.1, where $E(r_s)$ is the expected return on the spot asset, r is the risk-free rate, and $E(\phi)$ is the risk premium—the excess of expected return over the risk-free rate.

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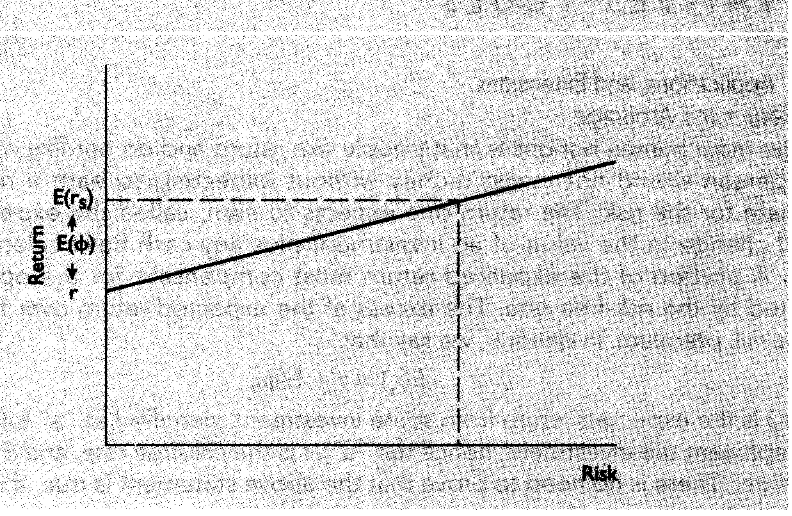
[1]In this context, "all other things being equal" means that the comparisons have not been distorted by differences in the number of shares outstanding or the amount of financial leverage.

[2]The concept of the risk-free rate and opportunity cost is well illustrated by the Biblical parable about the wealthy man who entrusted three servants to manage some of his money. Two of the servants earned 100 percent returns, while the third buried the money and returned only the principal sum. The wealthy man was infuriated that the third servant had not even earned the risk-free interest rate by putting the money in the bank, whereupon he reallocated the funds to one of the other servants' portfolios. The third servant, who was summarily discharged, evidently was not destined for a career as an investment manager (*Matthew* 25:14–30).

FIGURE 1.1 **Risk-Return Trade-Off**



Note that we have not identified how risk is measured. You might recall risk measures such as standard deviation and beta. At this point, we need not be concerned with the specific measure of risk. The important point is the positive relationship between risk and expected return, known as the risk-return trade-off. The risk-return trade-off arises because all investors seek to maximize expected return subject to a minimum level of risk. If a stock moves up the line into a higher risk level, some investors will find it too risky and will sell the stock, which will drive down its price. New investors in the stock will expect to earn higher returns by virtue of paying a lower price for the stock.

The financial markets are very effective at discriminating among firms with different risk levels. Firms with low risk will find capital plentiful and inexpensive. Firms with high risk may have trouble raising capital and will pay dearly. Markets that do a good job of pricing the instruments trading therein are said to be efficient, and the assets are said to be priced at their theoretical fair values.

Market Efficiency and Theoretical Fair Value

Market efficiency is the characteristic of a market in which the prices of the instruments trading therein reflect their true economic values to investors. In an efficient market, prices fluctuate randomly and investors cannot consistently earn returns above those that would compensate them for the level of risk they assume.

The idea that an asset has a "true economic value" is a particularly appealing concept. It suggests that somewhere out there is the *real* value of the asset. If we could determine the real value, we could perhaps make lots of money buying when the asset is priced too low and selling when it is priced too high. But finding the true economic value requires a model of how the asset is priced.

In this book we shall call the true economic value of the asset its *theoretical fair value*. There are many models that give the theoretical fair values of assets. You have probably already heard of the Capital Asset Pricing Model and perhaps the Arbitrage Pricing Theory. Derivatives also have theoretical fair values and in this book a great deal of emphasis is placed on determining the theoretical fair value of a derivative contract. Of course, these models and their respective values are correct only if the underlying market is efficient. Fortunately, there is considerable statistical evidence supporting the notion that financial markets are efficient. This is not surprising. Market efficiency is a natural consequence of rational and knowledgeable investor behavior in markets in which information spreads rapidly and inexpensively. We should be surprised if financial markets were highly inefficient.

DERIVATIVES TOOLS

Concepts, Applications, and Extensions
Risk and Return and Arbitrage

One of the most human notions is that people like return and do not like risk. It follows that a rational person would not invest money without expecting to earn a return sufficient to compensate for the risk. The return one expects to earn, called the *expected return*, is the expected change in the value of an investment plus any cash flows relative to the amount invested. A portion of the expected return must compensate for the opportunity cost, as represented by the risk-free rate. The excess of the expected return over the risk-free rate is called the *risk premium*. In general, we say that

$$E(r_s) = r + E(\phi),$$

where $E(r_s)$ is the expected return from some investment identified as "s" (oftentimes a stock is used to represent the investment, hence the "s"), r is the risk-free rate, and $E(\phi)$ is the expected risk premium. There is no need to prove that the above statement is true. If it were not, people would be irrational.

But a part of the equation is somewhat vague. What does the expected risk premium consist of? How large is it? What makes it change? What risk is important and what risk, if any, is not important? Financial economists have appealed to the *Capital Asset Pricing Model*, or CAPM, for answers. In the CAPM, the expected risk premium is replaced by something more specific. The expected return is written as follows:

$$E(r_s) = r + [E(r_m) - r]\,\beta_s,$$

where $E(r_m)$ is the expected return on the market portfolio, which is the combination of all risky assets, and β_s is called the asset's beta. The beta is a measure of the risk that an investor cannot avoid, which is the risk that the asset contributes to the market portfolio. The CAPM assumes that individuals diversify away as much risk as possible and hold the market portfolio. Thus, the only risk that matters is the risk that a given asset contributes to a diversified portfolio. As noted, investors hold the market portfolio and combine it with the risk-free asset or leverage it by borrowing at the risk-free rate so that the overall risk will be at the desired level. Hence, from the CAPM we get our first look at what risk management means: to force the actual portfolio risk to equal the desired portfolio risk.

The CAPM is a controversial theory. Whether it holds true in practice cannot be verified. Nonetheless, it makes considerable sense. Variations of the CAPM and more complex models do exist, but understanding and accepting the CAPM is more than enough background to understanding derivatives. Yet understanding the CAPM is not completely necessary for understanding derivatives. It does indeed help to understand how risk is accounted for. But so much of what matters in understanding derivatives is understanding how they can be used to eliminate risk. With risk out of the picture, all one really needs to understand is *arbitrage*.

Arbitrage is a condition resulting from the fact that two identical combinations of assets are selling for different prices. An investor who spots such an opportunity will buy the lower-priced combination and sell the higher-priced combination. Because the combinations of assets perform identically, the performance of one combination hedges the performance of the other so that the risk is eliminated. Yet one was purchased for one price and the other was sold for a higher price. Some people refer to this as a money tree or money machine. In other words, you get money for doing nothing.

A world of rational investors is a world in which arbitrage opportunities do not exist. It is often said that in such a world it would be impossible to walk down the street and find a $100 bill on the ground. If such a bill were ever there, someone would surely have already picked it up. Even good citizens and humanitarians would probably pick it up, hoping to find the owner or planning to give it to a charity. Of course, we know there is a possibility that we might find a $100 bill on the ground. But we do not expect to find one because people are not careless with large amounts of money, and if someone happens to be careless, it is unlikely the money will still be there by the time we arrive. And so it is in financial markets. People are not careless with their money. They are particularly careful, and they do not offer arbitrage opportunities. In fact, they work hard at understanding how to avoid offering arbitrage opportunities. And if anyone does offer an arbitrage opportunity, it will be snapped up quickly.

Studying this book will help you avoid offering arbitrage opportunities. And if someone carelessly offers an arbitrage opportunity, you will know how to take it.

Thus, as we weave our way through the world of derivatives, we should keep in mind that, by and large, the underlying financial markets are efficient. Although this book presents numerous strategies for using derivatives, all of them assume that the investor has already developed expectations about the direction of the market. Derivative strategies show how to profit if those expectations prove correct and how to minimize the risk of loss if they prove wrong. These strategies are methods for managing the level of risk and thus should be considered essential tools for survival in efficient markets.

FUNDAMENTAL LINKAGES BETWEEN SPOT AND DERIVATIVE MARKETS

So far we have not established a formal connection between spot and derivative markets. Instruments such as options, forwards, and futures are available for the purchase and sale of spot market assets, such as stocks and bonds. The prices of the derivatives are related to those of the underlying spot market instruments through several important mechanisms. Chapters 3, 4, 5, and 9 examine these linkages in detail; nevertheless, a general overview of the process here will be beneficial.

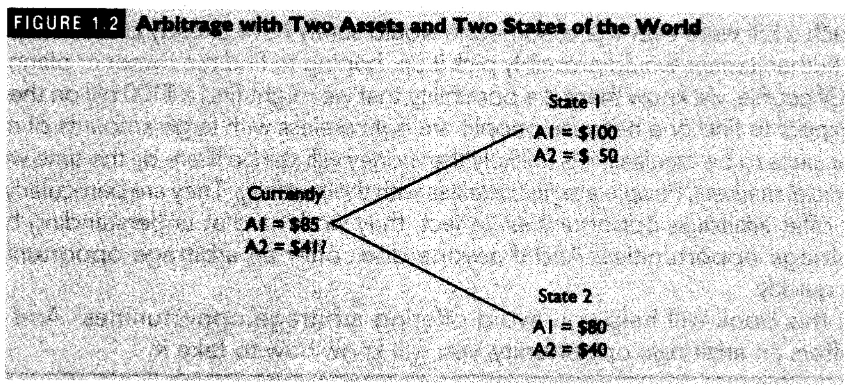
Arbitrage and the Law of One Price

Arbitrage is a type of transaction in which an investor seeks to profit when the same good sells for two different prices. The individual engaging in the arbitrage, called the arbitrageur, buys the good at the lower price and immediately sells it at the higher price. Arbitrage is an attractive strategy for investors. Thousands of individuals devote their time to looking for arbitrage opportunities. If a stock sells on one exchange at one price and on another at a different price, arbitrageurs will go to work buying at the low price and selling at the high price. The low price will be driven up and the high price driven down until the two prices are equal.

In your day-to-day life, you make many purchases and sales. Sometimes you encounter the same good selling for two different prices; for example, a computer from a mail-order discount house may cost less than the same computer at a local computer store. Why is there a difference? The store may offer longer warranties, localized service, and other conveniences not available through the discounter. Likewise, a pair of running shoes purchased at a local discounter may be cheaper than the same one purchased at a sporting goods store, where you pay extra for service and product knowledge. Where real differences exist between identical goods, the prices will differ.

But sometimes the differences appear real when they actually are not. For example, suppose there are two possible outcomes that might occur. We call these possible outcomes states. Look at the outcomes for two assets illustrated in Figure 1.2: A1 and A2. If state 1 occurs, asset A1 will be worth $100, while if state 2 occurs, asset A1 will be worth $80. In state 1 asset A2 will be worth $50, and in state 2 asset A2 will be worth $40. It should be obvious that asset A1 is equivalent to two shares of asset A2. Or in other words, by buying two shares of asset A2, you could obtain the same outcomes as buying one share of asset A1.

FIGURE 1.2 Arbitrage with Two Assets and Two States of the World



Now, suppose asset A1 is selling for $85. What should be the price of asset A2? Suppose asset A2 is $41. Then you could buy two shares of asset A2, paying $82, and sell short one share of asset A1 by borrowing the share from your broker. The short sale of asset A1 means you will receive its price, $85, up front, for a net cash flow of +$3 (= +$85 − $82). Then when the actual state is revealed, you can sell your two shares of asset A2 and generate exactly the amount of cash needed to cover your short sale position in asset A1. Thus, there is no risk to this transaction, and yet you received a net cash flow of $3 up front. This is like a loan in which you borrow $3 and do not have to pay back anything. Obviously, everyone would do this, which would push up the price of asset A2 and push down the price of asset A1 until the price of asset A1 was exactly equal to two times the price of asset A2.

The rule that states that these prices must be driven into line in this manner is called the law of one price. The law of one price does not mean that the price of asset A2 must equal the price of asset A1. Rather it states that equivalent combinations of financial instruments must have a single price. Here the combination of two shares of asset A2 must have the same price as one share of asset A1.

Markets ruled by the law of one price have the following four characteristics:

- Investors always prefer more wealth to less.
- Given two investment opportunities, investors will always prefer one that performs at least as well as the other in all states and better in at least one state.
- If two investment opportunities offer equivalent outcomes, they must have equivalent prices.
- An investment opportunity that produces the same return in all states is risk-free and must earn the risk-free rate.

In later chapters, we shall see these rules in action.

In an efficient market, violations of the law of one price should never occur. But occasionally prices get out of line, perhaps through momentary oversight. Arbitrage is the mechanism that keeps prices in line. To make intelligent investment decisions, we need to learn how arbitrage transactions are made, which we shall do in later chapters.

The Storage Mechanism: Spreading Consumption across Time

Storage is an important linkage between spot and derivative markets. Many types of assets can be purchased and stored. Holding a stock or bond is a form of storage. Even making a loan is a form of storage. One can also buy a commodity, such as wheat or corn, and store it in a grain elevator. Storage is a form of investment in which one defers selling the item today in anticipation of selling it at a later date. Storage spreads consumption across time.

Because prices constantly fluctuate, storage entails risk. Derivatives can be used to reduce that risk by providing a means of establishing today the item's future sale price. This suggests that the risk entailed in storing the item can be removed. In that case, the overall investment should offer the risk-free rate. Therefore, it is not surprising that the prices of the storable item, the derivative contract, and the risk-free rate will all be related.

Delivery and Settlement

Another important linkage between spot and derivative markets is delivery and settlement. At expiration, a forward or futures contract calls for either immediate delivery of the item or a cash payment of the same value. Thus, an expiring forward or futures contract is equivalent to a spot transaction. The price of the expiring contract, therefore, must equal the spot price. Though options differ somewhat from forwards and futures at expiration, these instruments have an unambiguous value at expiration that is determined by the spot price.

Few derivative traders hold their positions until the contracts expire.[3] They use the market's liquidity to enter into offsetting transactions. Nonetheless, the fact that delivery or an equivalent cash payment will occur on positions open at expiration is an important consideration in pricing the spot and derivative instruments.

The foregoing properties play an important role in these markets' performance. Derivative and spot markets are inextricably linked. Nonetheless, we have not yet determined what role derivative markets play in the operations of spot markets.

THE ROLE OF DERIVATIVE MARKETS

Risk Management

Because derivative prices are related to the prices of the underlying spot market goods, they can be used to reduce or increase the risk of owning the spot items. Derivative market participants seeking to reduce their risk are called hedgers. Derivative market participants seeking to increase their risk are called speculators.

For example, buying the spot item and selling a futures contract or call option reduces the investor's risk. If the good's price falls, the price of the futures or option contract will also fall. The investor can then repurchase the contract at the lower price, effecting a gain that can at least partially offset the loss on the spot item. This type of transaction is known as a hedge.

As we noted earlier, investors have different risk preferences. Some are more tolerant of risk than others. All investors, however, want to keep their investments at an acceptable risk level. Derivative markets enable those investors who wish to reduce their risk to transfer it to those wishing to increase it. We call these latter investors speculators. Because these markets are so effective at reallocating risk among investors, no one need assume an uncomfortable level of risk. Consequently, investors are willing to supply more funds to the financial markets. This benefits the economy, because it enables more firms to raise capital and keeps the cost of that capital as low as possible.

As noted, on the other side of hedging is speculation. Unless a hedger can find another hedger with opposite needs, the hedger's risk must be assumed by a speculator. Derivative markets provide an alternative and efficient means of speculating. Instead of trading the underlying stocks or bonds, an investor can trade a derivative contract. Many investors prefer to speculate with derivatives rather than with the underlying securities. The ease with which speculation can be done using derivatives in turn makes it easier and less costly for hedgers.

We would be remiss if we left it at that, however, for speculation is controversial. Derivative markets have taken much criticism from outsiders, including accusations that their activities are tantamount to legalized gambling. We shall look at this point in a later section.

Price Discovery

Forward and futures markets are an important source of information about prices. Futures markets, in particular, are considered a primary means for determining the spot price of an asset. This should seem

[3]On derivative contracts that do not call for delivery at expiration, but specify that an economically equivalent cash payment be made, positions are more likely to be held to expiration.

unusual, since a spot market for the asset must exist, but for many assets on which futures trade, the spot market is large and fragmented. Gold, oil, and commodities trade at different places and at different times. Within each asset's class, there are many varieties and quality grades. Hence, there are many potential candidates for the "spot" price of an asset. The futures market assembles that information into a type of consensus, reflecting the spot price of the particular asset on which the futures contract is based. The price of the futures contract that expires the earliest, referred to as the nearby contract, is often treated as the spot price.

Futures and forward prices also contain information about what people expect future spot prices to be. As we shall see later, spot prices contain this same information, but it may be harder to extract that information from the spot market than from the futures market. Moreover, in almost all cases, the futures market is more active and, hence, information taken from it is often considered more reliable than spot market information. While a futures or forward price should not be treated as an expected future spot price, a futures or forward price does reflect a price that a market participant could lock in today in lieu of accepting the uncertainty of the future spot price.

Hence, futures and forward markets are said to provide price discovery. Options markets do not directly provide forecasts of future spot prices. They do, however, provide valuable information about the volatility and, hence, the risk of the underlying spot asset.

Operational Advantages

Derivative markets offer several operational advantages. First, they entail lower transaction costs. This means that commissions and other trading costs are lower for traders in these markets. This makes it easy and attractive to use these markets either in lieu of spot market transactions or as a complement to spot positions.

Second, derivative markets often have greater liquidity than the spot markets. Although spot markets generally are quite liquid for the securities of major companies, they cannot always absorb some of the large dollar transactions without substantial price changes. In some cases, one can obtain the same levels of expected return and risk by using derivative markets, which can more easily accommodate high-volume trades. This higher liquidity is at least partly due to the smaller amount of capital required for participation in derivative markets. Returns and risks can be adjusted to any level desired, but because less capital is required, these markets can absorb more trading.

Third, as noted earlier, derivative markets allow investors to sell short in an easier manner. Securities markets impose several restrictions designed to limit or discourage short selling that are not applied to derivative transactions. Consequently, many investors sell short in these markets in lieu of selling short the underlying securities.

Market Efficiency

Spot markets for securities probably would be efficient even if there were no derivative markets. A few profitable arbitrage opportunities exist, however, even in markets that are usually efficient. The presence of these opportunities means that the prices of some assets are temporarily out of line with what they should be. Investors can earn returns that exceed what the market deems fair for the given risk level.

As noted earlier, there are important linkages among spot and derivative prices. The ease and low cost of transacting in these markets facilitate the arbitrage trading and rapid price adjustments that quickly eradicate these profit opportunities. Society benefits because the prices of the underlying goods more accurately reflect the goods' true economic values.

CRITICISMS OF DERIVATIVE MARKETS

As noted earlier, derivative markets allow the transfer of risk from those wanting to remove or decrease it to those wanting to assume or increase it. These markets require the presence of speculators willing to assume risk to in order to accommodate the hedgers wishing to reduce it. Most speculators do not actually deal in the underlying goods and sometimes are alleged to know nothing about them. Consequently, these speculators have been characterized as little more than gamblers.

This view is a bit one-sided and ignores the many benefits of derivative markets. More important, it suggests that these markets siphon capital into wildly speculative schemes. Nothing could be further from the truth. Unlike financial markets, derivative markets neither create nor destroy wealth—they merely provide a means to transfer risk. For example, stock markets can create wealth. Consider a firm with a new idea that offers stock to the public. Investors buy the stock, and the firm uses the capital to develop and market the idea. Customers then buy the product or service, the firm earns a profit, the stock price increases, and everyone is better off. In contrast, in derivative markets one party's gains are another's losses. These markets put no additional risk into the economy; they merely allow risk to be passed from one investor to another. More important, they allow the risk of transacting in real goods to be transferred from those not wanting it to those willing to accept it.

An important distinction between derivative markets and gambling is in the benefits provided to society. Gambling benefits only the participants and perhaps a few others who profit indirectly. The benefits of derivatives, however, extend far beyond the market participants. Derivatives help financial markets become more efficient and provide better opportunities for managing risk. These benefits spill over into society as a whole.

MISUSES OF DERIVATIVES

Derivatives have occasionally been criticized for having been the source of large losses by some corporations, investment funds, state and local governments, nonprofit investors, and individuals. Are derivatives really at fault? Is electricity to be faulted when someone with little knowledge of it mishandles it? Is fire to be blamed when someone using it becomes careless?

There is little question that derivatives are powerful instruments. They typically contain a high degree of leverage, meaning that small price changes can lead to large gains and losses. Though this would appear to be an undesirable feature of derivatives, it actually is what makes them most useful in providing the benefits discussed earlier. These are points we shall study later. At this time, however, you should recognize that to use derivatives without having the requisite knowledge is dangerous. That is all the more reason why you should be glad you have chosen to study the subject.

Having acquired that knowledge, however, does not free you of the responsibility to act sensibly. To use derivatives in inappropriate situations is dangerous. The temptation to speculate when one should be hedging is a risk that even the knowledgeable often succumb to. Having excessive confidence in one's ability to forecast prices or interest rates and then acting on those forecasts by using derivatives can be extremely risky. You should never forget what we said about efficient markets. Regrettably, in recent years many individuals have led their firms down the path of danger and destruction by forgetting these points, with the consequence that derivatives and not people are often blamed.

Fortunately, derivatives are normally used by knowledgeable persons in situations where they serve an appropriate purpose. We hear far too little about the firms and investors who saved money, avoided losses, and restructured their risks successfully.

DERIVATIVES AND YOUR CAREER

It is tempting to believe that derivatives are but an interesting subject for study. You might feel that you would someday want to buy an option for your personal investment portfolio. You might think that you are unlikely to encounter derivatives in your career in business. That is simply not true.

As we noted earlier, the primary use of derivatives is in risk management. Businesses, by their very nature, face risks. Some of those risks are acceptable; indeed a business must assume some type of risk or there is no reason to be in business. But other types of risks are unacceptable and should be managed, if not eliminated. For example, a small furniture manufacturer may borrow money from a bank at a rate that will be adjusted periodically to reflect current interest rates. The furniture manufacturer is in the business of making money off the furniture market. It is not particularly suited to forecasting interest rates. Yet interest rate increases could severely hamper its ability to make a profit from its furniture business. If that firm sells its products in foreign countries, it may face significant foreign exchange risk. If the raw materials it purchases and the energy it consumes are subject to uncertain future prices, as they surely are, the firm faces additional risks, all having the potential to undermine its success in its main line of business.

It was but a few years ago that a small firm would not be expected to use derivatives to manage its interest rate or foreign exchange risk, nor would it be able to do so if it wanted. The minimum sizes of transactions were too large. Times have changed and smaller firms are now more able to use derivatives.

If your career takes you into investment management, you will surely encounter derivatives. Those in public service who manage the assets of governments are finding numerous applications of derivatives. Those responsible for the commodities and energy purchased by firms will encounter situations where derivatives are or can be used. In short, derivatives are becoming commonplace and are likely to be even more so for the foreseeable future.

By taking a course and/or reading this book on derivatives, you are making the first step toward obtaining the tools necessary to understand the nature and management of risk, a subject that lies at the very heart of a business.

SOURCES OF INFORMATION ON DERIVATIVES

The derivative markets have become so visible in today's financial system that virtually any publication that covers the stock and bond markets contains some coverage of derivatives. There are a variety of specialized trade publications, academic and professional journals, and Internet sites provided by a number of companies and governmental agencies. We maintain a Web site containing many of these links as well as links to collections of links maintained by others. Access the site through the *Author Updates* link on the book's Web site, www.academic.cengage.com/finance/chance.

BOOK OVERVIEW

We provide here a brief overview of the book, including the new features of the Seventh Edition.

Organization of the Book

This book is divided into three main parts. First there is an introductory chapter, which gives an overview of the book's subject. Then Part I, consisting of Chapters 2–7, covers options. Chapter 2 introduces the basic characteristics of options and their markets. Chapter 3 presents the fundamental principles of pricing options. These principles are often called boundary conditions, and while we do tend to think of them as fundamental, they are nonetheless quite challenging. Chapter 4 presents the simple binomial model for pricing options. Chapter 5 covers the Black-Scholes-Merton model, which is the premier tool for pricing options and for which a Nobel Prize was awarded in 1997. Chapters 6 and 7 cover option trading strategies.

Part II covers forwards, futures, and swaps. It begins with Chapter 8, which introduces the basic characteristics of forward and futures markets. Chapter 9 presents the principles for pricing forwards, futures, and options on futures contracts. Chapter 10 covers futures arbitrage strategies, which is the primary determinant of futures prices. Chapter 11 covers various futures trading strategies. Chapter 12 is devoted to swaps, including interest rate, currency, and equity swaps.

Part III deals with various advanced topics, although one should not get the impression that the material is particularly complex. Chapter 13 deals with interest rate derivatives, such as forward rate agreements, interest rate options, and swaptions. Chapter 14 covers some advanced derivatives and strategies, which are mostly extensions of previous topics and strategies. Chapters 15 and 16 deal with risk management. Chapter 15 covers quantitative risk management, emphasizing such topics as Value at Risk, delta hedging, and managing credit risk. Chapter 16 is more qualitative and focuses on the issues that must be addressed in an organization so that risk management is properly conducted. You will have the opportunity to learn how risk management is done well in organizations and how it is done poorly.

Key Features of the Book

Some key features of the book are:

- An emphasis on practical application of theory; all ideas and concepts are presented with clear illustrations. You never lose touch with the real world.
- A minimal use of technical mathematics. While financial derivatives is unavoidably a technical subject, calculus is not necessary for learning the material at this level. (Note: Some calculus is used in appendix, but it is not essential for understanding that material.)
- A balanced emphasis on strategies and pricing.
- A liberal use of illustrations. The book contains over 100 figures and is supported with over 100 tables.
- Over 330 end-of-chapter questions and problems that allow you to test your skills (solutions keyed to chapter sub-headings are available to adopting instructors).
- Downloadable software: A Windows program and various Excel spreadsheets are available online at www.academic.cengage.com/aise. Throughout the book there are sections called Software Demonstrations that contain explicit illustrations of how to use the software.
- Appendices containing lists of formulas and references.
- A comprehensive index, of course.
- A PowerPoint® presentation, which is available for instructors adopting the book. Some of the other PowerPoint presentations that we have seen accompanying finance textbooks are mostly just outlines; this book's presentations, however, contain much more detail, and are available for download at **www.academic.cengage.com/aise.**

Our focus in this book is on making theory work in practice. All points are illustrated as much as possible using practical situations. When strategies are covered, readers learn the theory, examine the algebraic equations that describe what is happening, and observe the results with either a table or graph.

Specific New Features of the Seventh Edition

For those familiar with previous editions, the following are new features:

- An update of contemporary market data references
- Technical note references within chapters that direct students to more details on the Web site. This feature allows more complex materials to be available to those faculty and students who desire to explore the book's subject in more depth without distracting others. In some cases technical appendices that formerly appeared in the book have been moved to the Web.
- Major revisions to Chapters 9 through 11 that improve the coverage of futures and forward markets. This material now brings the arbitrage transactions that support futures and forward pricing closer to the pricing material.

- Stock and option price references that have all been converted from fractions—such as 3/8—to dollars and cents
- Expanded coverage of credit derivatives in Chapter 15

Previous users of the book will also note in particular that the options examples are no longer associated with the company AOL. Due to the possibility of mergers and other major changes that could occur to any company over time, we have elected to no longer identify our examples as those of an actual company. In this edition, the prices you see are the same as in the previous edition, but the authors have taken the liberty of creating their own (self-named) hypothetical company and guarantee that it will not be acquired by another hypothetical company!

Use of the Book

The ideal way to use this book, and almost all finance textbooks, is in a two-semester course. A full academic year gives an excellent opportunity to cover the subject matter without flying at breakneck speed. Each semester can consist of eight chapters, leaving some time for quizzes, exams, and other in-class activities, such as watching a video or engaging in a trading exercise. If, however, this book is used for only a one-semester course, instructors should find the material sufficiently flexible for picking and choosing chapters. There is a tendency, however, for one-semester courses to just cover the chapters in the order in which they appear. Our own recommendation is that a one-semester course should be sure to include swaps. The swap is the most widely used derivative and the one most likely to be encountered by those who go out into the corporate world. Thus, the instructor might wish to make a special effort to cover Chapter 12, which would probably not be covered if the syllabus just followed the sequence of chapters. In addition to swaps, a one-semester course should probably include interest rate options, which are also likely to be encountered in the corporate world. To make room for these topics, the instructor might need to de-emphasize futures and possibly even cut down on the coverage of option strategies. Chapters 14, 15, and 16, therefore, are the lowest-priority chapters that should be forced into a one-semester course.

Although the primary audience is the university-level undergraduate, this book has been widely used at the MBA level, including at some very prestigious universities' MBA programs. Instructors should not hesitate to adapt the book to an MBA course. The book has also been used in corporate training programs.

Questions and Problems

1. Why is delivery important if so few futures contracts end in delivery?
2. Suppose you are shopping for a new automobile. You find the same car at two dealers but at different prices. Is the law of one price being violated? Why or why not?
3. What is an efficient market? Why do efficient markets benefit society?
4. Define arbitrage and the law of one price. What role do they play in our market system? What do we call the "one price" of an asset?
5. Why is speculation controversial? How does it differ from gambling?
6. Distinguish between business risk and financial risk.
7. What is the difference between an investor who is risk neutral and one who is risk averse?
8. Distinguish between real assets and financial assets.
9. Explain the concept of a risk-return trade-off.
10. What are the components of the expected return?

11. An option dealer needs to finance the purchase of a security and holds an inventory of U.S. Treasury bills. Explain how the dealer can use the repo market for financing the security purchase.
12. Contrast dollar return and percentage return. Be sure to identify which return is more useful when comparing investments.
13. What is storage? Why is it risky? What role does it play in the economy?
14. Assume that you have an opportunity to visit a civilization in outer space. Its society is at roughly the same stage of development as U.S. society is now. Its economic system is virtually identical to that of the United States, but derivative trading is illegal. Compare and contrast this economy with the U.S. economy, emphasizing the differences due to the presence of derivative markets in the latter.
15. What are the three ways in which derivatives can be misused?
16. What are the major functions of derivative markets in an economy?

PART 1

Options

2

STRUCTURE OF OPTIONS MARKETS

In Chapter 1 we introduced the concept of an option, a contract between two parties—a buyer and a seller, or writer—in which the buyer purchases from the writer the right to buy or sell an asset at a fixed price. As in any contract, each party grants something to the other. The buyer pays the seller a fee called the premium, which is the option's price. The writer grants the buyer the right to buy or sell the asset at a fixed price.

An option to buy an asset is a call option. An option to sell an asset is a put option. The fixed price at which the option buyer can either buy or sell the asset is called the exercise price or strike price, or sometimes the striking price. In addition, the option has a definite life. The right to buy or sell the asset at a fixed price exists up to a specified expiration date.

Options are often encountered in everyday life. For example, a rain check offered by a store to allow you to return and purchase a sale item that is temporarily out of stock is an option. You return to the store and buy the item, if it is really worth it. Or you can throw the rain check away. You could even give (or sell) it to someone else. A coupon clipped from the newspaper that allows you to buy an item for a special price at any time up to an expiration date is also an option. Suppose you plan to purchase a deeply discounted airline ticket. You are afraid your plans might change, and the airline says the ticket at that price is nonrefundable. For just $75 more, you can obtain the right to cancel the ticket at the last minute. If you pay the extra $75 for the right to cancel, you have just purchased an option. Finally, there is a good chance you are taking a college or university course right now, and you probably hold a valuable option: your right to drop the course up to a specific date. That right is paid for automatically with your tuition payment. On a certain date later in the term, you will decide whether to continue with the course or drop it. This is certainly a valuable option, but one we hope you will not use, at least not in this course.

In each example, you hold the right to do something. You will exercise that right if it turns out to be worth it to you. Although that is the essence of an option, options on securities and other assets do have a few more complicating features. First however, let us take a look at how options markets developed.

DEVELOPMENT OF OPTIONS MARKETS

There are plenty of examples of options in everyday life. Historians and archaeologists have even discovered primitive options. Though these arrangements may resemble modern options, the current system of options markets traces its origins to the nineteenth century, when puts and calls were offered on shares of stock. Little is known about the options world of the 1800s other than that it was fraught with corruption.

Then, in the early 1900s, a group of firms calling itself the Put and Call Brokers and Dealers Association created an options market. If someone wanted to buy an option, a member of the association would find a seller willing to write it. If the member firm could not find a writer, it would write the option itself. Thus, a member firm could be either a broker—one who matches buyer and seller—or a dealer—one who actually takes a position in the transaction.

Although this over-the-counter options market was viable, it suffered from several deficiencies. First, it did not provide the option holder the opportunity to sell the option to someone else before it expired. Options were designed to be held all the way to expiration, whereupon they were either exercised or allowed to expire. Thus, an option contract had little or no liquidity. Second, the writer's performance was guaranteed only by the broker-dealer firm. If the writer or the Put and Call Brokers and Dealers Association member firm went bankrupt, the option holder suffered a credit loss.[1] Third, the cost of transacting was relatively high, due partly to the first two problems.

In 1973, a revolutionary change occurred in the options world. The Chicago Board of Trade, the world's oldest and largest exchange for the trading of commodity futures contracts, organized an exchange exclusively for trading options on stocks. The exchange was named the Chicago Board Options Exchange (CBOE). It opened its doors for call option trading on April 26, 1973, and the first puts were added in June 1977.

The CBOE created a central marketplace for options. By standardizing the terms and conditions of option contracts, it added liquidity. In other words, an investor who had previously bought or sold an option could go back into the market prior to its expiration and sell or buy the option, thus offsetting the original position. Most importantly, however, the CBOE added a clearinghouse that guaranteed to the buyer that the writer would fulfill his or her end of the contract. Thus, unlike in the over-the-counter market, option buyers no longer had to worry about the credit risk of the writer. This made options more attractive to the general public.

Since that time, several stock exchanges and almost all futures exchanges have begun trading options. Fueled by the public's taste for options, the industry grew tremendously until the great stock market crash of 1987. Hit by the shock of the crash, many individual investors who had formerly used options stayed away, and volume recovered to its 1987 level in 1997 and has been rising. In 2004, total option contract volume exceeded one billion for the first time.

Although institutional trading on the options exchanges remained fairly strong after the crash, a concurrent trend forced the exchanges to address a new competitive threat: the revival of the over-the-counter options markets. In the early 1980s, many corporations began to use currency and interest rate swaps to manage their risk. These contracts, which we briefly mentioned in Chapter 1 and shall cover in more detail in Chapter 12, are private transactions that are tailored to the specific needs of the two parties. They are subject to credit risk in that a party could default, leaving the opposite party holding a claim that had to be pursued in bankruptcy courts. As it turned out, however, these claims were few and far between, and the market functioned exceptionally well. Soon thereafter, firms began to create other types of over-the-counter contracts, such as forwards, and, as expected, options began to be used as well. Because of the large minimum size of each transaction and the credit risk, however, the general public is unable to participate in this new, revived over-the-counter market. The growth in this institutional over-the-counter market has placed severe pressures on the options exchanges. By the early 1990s the exchanges were trying to become more innovative to win back institutional trading and to stimulate the public's interest in options. These trends, however, should not suggest that options are fading in popularity; in fact they are more popular than ever with corporations and financial institutions, but the growth is concentrated in the over-the-counter market.

CALL OPTIONS

A call option is an option to buy an asset at a fixed price—the exercise price. Options are available on many types of assets, but for now we shall concentrate on stock options.[2] Consider the following example: On August 1, 2005, several exchanges offered options on the stock of Microsoft. One particular call option had an exercise price of $27.5 and an expiration date of September 16. Microsoft stock had a price of $25.92. The

[1]The individual or firm could, of course, pursue costly legal remedies.

[2]As we noted in Chapter 1, derivatives are created not only on assets, but also on futures contracts and the weather, neither of which would be considered an asset.

buyer of this option received the right to buy the stock at any time up through September 16 at $27.5 per share. The writer of that option therefore was obligated to sell the stock at $27.5 per share through September 16 whenever the buyer wanted it. For this privilege, the buyer paid the writer the premium, or price, of $0.125.

Why would either party have entered into the call option contract? The call option buyer would not have done so to immediately exercise the option, because the stock could be bought in the market for $25.92, which was less than the exercise price of $27.5. The call option buyer must have expected that the stock's price would rise above $27.5 before the option expired. Conversely, the call writer expected that the stock price would not get above $27.5 before the option expired. The call buyer and writer negotiated the premium of $0.125, which can be viewed as the call buyer's wager on the stock price going above $27.5 by September 16. Alternatively, either the call buyer or writer may have been using the option to protect a position in the stock—a strategy we mentioned in Chapter 1 called hedging.

Suppose that immediately after a call is purchased, the stock price increases. Because the exercise price is constant, the call option is now more valuable. New call options with the same terms will sell for higher premiums. Therefore, older call options with the same expiration date and exercise price must also sell for higher premiums. Similarly, if the stock price falls, the call's price also will decline. Clearly the buyer of a call option has bullish expectations about the stock.

A call in which the stock price exceeds the exercise price is said to be in-the-money. As we shall see in Chapter 3, however, in-the-money calls should not necessarily be exercised prior to expiration. If the stock price is less than the exercise price, the call option is said to be out-of-the-money. Out-of-the-money calls should never be exercised. We shall explore these points more thoroughly in Chapter 3. If the stock price equals the exercise price, the option is at-the-money.

PUT OPTIONS

A put option is an option to sell an asset, such as a stock. Consider the put option on Microsoft stock on August 1, 2005, with an exercise price of $27.5 per share and an expiration date of September 16. It allowed the put holder to sell the stock at $27.5 per share any time up through September 16. The stock currently was selling for $25.92. Therefore, the put holder could have elected to exercise the option, selling the stock to the writer for $27.5 per share. The put holder may, however, have preferred to wait and see if the stock price fell further below the exercise price. The put buyer expected the stock price to fall, while the writer expected it to remain the same or rise.

The put option buyer and writer negotiated a premium of $1.70, which the buyer paid to the writer. The put option premium can be viewed as the buyer's wager that the stock price would not rise above $27.5 per share by September 16. The put writer accepted the premium because it was deemed to be fair compensation for the willingness to buy the stock for $27.5 any time up through September 16. As in the case of call options, either the put option buyer or the writer might have been using the put to hedge a position in the stock.

Since the put allows the holder to sell the stock for a fixed price, a decrease in the stock price will make the put more valuable. Conversely, if the stock price increases, the put will be less valuable. It should be apparent that the buyer of a put has bearish expectations for the stock.

If the stock price is less than the exercise price, the put is said to be in-the-money. In Chapter 3, we shall see that it is sometimes, but not always, optimal to exercise an in-the-money put prior to expiration. If the stock price is more than the exercise price, the put is out-of-the-money. An out-of-the-money put should never be exercised. When the stock price equals the exercise price, the put is at-the-money.

OVER-THE-COUNTER OPTIONS MARKET

As noted earlier, there is now a rather large over-the-counter options market dominated by institutional investors. Chicago is no longer the center of the options industry. The scope of this market is worldwide. An option bought by an American corporation in Minnesota from the New York office of a Japanese bank, who in turn buys an offsetting option from the London office of a Swiss bank, would not at all be unusual. These contracts are entered into privately by large corporations, financial institutions, and sometimes even governments, and the option buyer is either familiar with the creditworthiness of the writer or has had the credit risk reduced by some type of collateral guarantee or other credit enhancement. Nonetheless, there is nearly always some credit risk faced by the buyers of these options. There are, however, several major advantages of this type of option.

The first advantage is that the terms and conditions of the options can be tailored to the specific needs of the two parties. For example, suppose the manager of a pension fund would like to protect the profit in the fund's portfolio against a general decline in the market. As we shall discuss in great detail in Chapter 6, the purchase of a put in which the holder of the portfolio can sell it to the option writer for a specific value on a certain date can assure the manager of a minimum return. Unfortunately, this type of transaction cannot always be accomplished on the options exchange. First, the options available on the exchange are based on certain stocks or stock indices. The manager would need an option on the specific portfolio, which might not match the indices on which options were available.[3] Second, the options on the exchange expire at specific dates, which might not match the manager's investment horizon. Third, even if options were available, there might not be enough liquidity to handle the large trades necessary to protect the entire portfolio. In the over-the-counter market, the manager can specify precisely which combination of stocks the option should be written on and when it should be exercised.[4] Although unusually large transactions could take some time to arrange, it is likely that most pension fund managers could get the desired transactions accomplished. Fourth, in the over-the-counter markets, options can be created on a wider range of instruments than just stocks. Options exist on bonds, interest rates, commodities, currencies, and many other types of assets as well as some instruments that are not even assets, such as the weather.

A second advantage is that the over-the-counter market is a private market in which neither the general public nor other investors, including competitors, need know that the transactions were completed. This does not mean that the transactions are illegal or suspicious. On the options exchange, a large order to buy puts could send a signal to the market that someone might have some bad news. This could send the market reeling as it worries about what impending information might soon come out.

Another advantage is that over-the-counter trading is essentially unregulated. Its rules are those of commonsense business honesty and courtesy. Institutions that do not conform would find themselves unable to find counterparties with which to trade. This largely unregulated environment means that government approval is not needed to offer new types of options. The contracts are simply created by parties that see mutual gain in doing business with each other. There are no costly constraints or bureaucratic red tape to cut through.

Clearly there are some disadvantages to over-the-counter trading, the primary one of which is that credit risk is higher and excludes many customers who are unable to establish their creditworthiness in this market. The credit risk problem is an important and highly visible contemporary issue in derivatives markets, and we shall discuss it more in later chapters. In addition to the credit risk problem, the sizes of the transactions in the over-the-counter market are larger than many investors can handle. It is not clear, however, that over-the-counter trading is any more or less costly than trading on the exchange.

The over-the-counter market is quite large, but because of the private nature of the transactions, gauging its size is difficult. The Bank for International Settlements (BIS) conducts semiannual surveys that attempt to provide some data. These surveys, however, take a long time to complete and are published with a considerable lag. The BIS's latest survey estimated that in December 2005, the outstanding notional principal, which is the amount of the underlying instrument, of options on interest rates, currencies, equities, and commodities was $40 trillion, with a market value of about $1.2 trillion.

Most of the options created on the over-the-counter market are not the traditional case of an option on an individual common stock. They tend to be options on bonds, interest rates, commodities, swaps, and foreign currencies and include many variations that combine options with other instruments. As noted earlier, a

[3]It would be wasteful to purchase an option on each security, even if one were available; the purchase would protect the portfolio against risk that is already eliminated by the portfolio diversification.

[4]Because the specific combination of stocks is called a *basket,* these types of options are called *basket options.*

significant number are created on equity portfolios or indices. Many are on foreign stock indices. The principles behind pricing and using options, however, are pretty much the same, whether the option is created on an options exchange or on the over-the-counter market. There are obvious variations to accommodate the different types of options. We shall cover many of these in later chapters. Most of the material on options, which comprises Chapter 2 through Chapter 7, however, will use examples that come from organized markets. Now let us take a look at how the organized options markets operate.

ORGANIZED OPTIONS TRADING

An exchange is a legal corporate entity organized for the trading of securities, options, or futures. It provides a physical facility and stipulates rules and regulations governing the transactions in the instruments trading thereon. In the options markets, organized exchanges evolved in response to the lack of standardization and liquidity of over-the-counter options. Over-the-counter options were written for specific buyers by particular sellers. The terms and conditions of the contracts, such as the exercise price and expiration date, were tailored for the parties involved. Organized exchanges filled the need for standardized option contracts wherein the exchange would specify the contracts' terms and conditions. Consequently, a secondary market for the contracts was made possible. This made options more accessible and attractive to the general public.

As a result of providing a trading facility, specifying rules and regulations, and standardizing contracts, options became as marketable as stocks. If an option holder wanted to sell the option before the expiration date or an option writer wished to get out of the obligation to buy or sell the stock, a closing transaction could be arranged at the options exchange. We shall examine these procedures in more detail in a later section.

The Chicago Board Options Exchange, the first organized options exchange, established the procedures that made options marketable. In addition, it paved the way for the American, Philadelphia, and Pacific Exchanges to begin option trading. The next several sections examine the CBOE's contract specifications.

Listing Requirements

The options exchange specifies the assets on which option trading is allowed. For stock options, the exchange's listing requirements prescribe the eligible stocks on which options can be traded. At one time, these requirements limited options listings to stocks of large firms, but these requirements have been relaxed, and more small firms' options are available for trading. The exchange also specifies minimum requirements that a stock must meet to maintain the listing of options on it. These requirements are similar to but slightly less stringent than those for the initial listing. In all cases, however, the exchange has the authority to make exceptions to the listing and delisting requirements.

All options of a particular type—call or put—on a given stock are referred to as an option class. For example, the Microsoft calls are one option class and the Microsoft puts are another. An option series is all the options of a given class with the same exercise price and expiration. For example, the Microsoft September 27.5 calls are a particular series, as are the Microsoft October 25 puts.

In recent years many options have been listed on more than one exchange. An options exchange determines whether options of a particular stock will be listed on its exchange. The company itself does not make this decision.

Contract Size

A standard exchange-traded stock option contract provides exposure to 100 individual stocks. Thus, if an investor purchases one contract, it actually represents options to buy 100 shares of stock. An exception to the standard contract size occurs when either a stock splits or the company declares a stock dividend. In that case, the number of shares represented by a standard contract is adjusted to reflect the change in the company's capitalization. For example, if a company declares a 15 percent stock dividend, the number of shares represented by an outstanding contract changes from 100 to 115. In addition, the exercise price is adjusted to

0.8696 (=1/1.15), rounded to the nearest eighth—or 0.875—of its former value. If a stock split or stock dividend results in the new number of shares being an even multiple of 100, holders of outstanding contracts are credited with additional contracts. For example, if the stock splits two-for-one, buyers and writers are credited with two contracts for every one formerly held. Remember, the exercise price will also be reduced to half of its previous value.

Contract sizes for options on indexes and certain other instruments are specified as a multiple. For example, an option on the S&P 100 index has a multiple of 100; an investor who buys one contract actually buys exposure to 100 times the index.

Exercise Prices

On options exchanges the exercise prices are standardized. Exchanges prescribe the exercise prices at which options can be written. Investors must be willing to trade options with the specified exercise prices. Of course, over-the-counter transactions can have any exercise price the two participants agree on.

The goal when establishing the exercise prices is to provide options that will attract trading volume. Most option trading is concentrated within options in which the stock price is close to the exercise price. Accordingly, exchange officials tend to list options in which the exercise prices surround but are close to the current stock price. They must use their judgment as to whether an exercise price is too far above or below the stock price to generate sufficient trading volume. If the stock price moves up or down, new exercise prices close to the stock price are added.

In establishing exercise prices of stock options, exchanges generally follow the rule that the exercise prices are in $2.50 intervals if the stock price is less than $25, in $5 intervals if the stock price is between $25 and $200, and in $10 intervals if the stock price is above $200. For index options, the exercise price intervals vary due to the wide range of the various indices. There are some exceptions to these rules; the very actively traded options have exercise prices closer together.

In 1993 the CBOE launched the FLEX (for flexible) option, a new type of option that represented a dramatic departure from the standardization of organized options markets. FLEX options can have any exercise price. In addition, there are other variations that we shall mention when we discuss expirations. FLEX options are available with a minimum face value of $10 million for index options and 250 contracts for options on individual stocks.

When a stock pays a dividend, the stock price typically falls by the present value of the dividend on the ex-dividend day, which is the day after the last day on which the purchaser of the stock is entitled to receive the upcoming dividend. Because call option holders do not receive dividends and benefit from increases in the stock price, and put option holders benefit from stock price decreases, the ex-dividend decrease in the stock price would arbitrarily hurt call holders and help put holders. In the old over-the-counter options market, options were dividend-protected. If the company declared a $1 dividend, the exercise price was reduced by $1. Since over-the-counter options were not meant to be traded, the frequent dividend adjustments caused no problems. For exchange-listed options, however, such dividend adjustments would have generated many nonstandard exercise prices. Thus, the exchanges elected not to adjust the exercise price when a cash dividend was paid. This is also true in the current over-the-counter market.

Expiration Dates

Expiration dates of over-the-counter options are tailored to the buyers' and writers' needs. On the options exchanges, each stock is classified into a particular expiration cycle. The expiration cycles are (1) January, April, July, and October; (2) February, May, August, and November; and (3) March, June, September, and December. These were called the *January, February,* and *March cycles.* The available expirations are the current month, the next month, and the next two months within the January, February, or March cycle to which the stock is assigned. For example, in early June, IBM, which is assigned to the January cycle, will have options expiring in June and July plus the next two months in the January cycle: October and the following January. When the June options expire, the August options will be added; when the July options expire, the

September options will be added; and when the August options expire, the April options will be added. Index options typically have expirations of the current and next two months.

The maturities of options on individual stocks go out about nine months with a few exceptions. LEAPS (Long-Term Equity Anticipation Shares) are options on certain stocks and indices that have expirations of up to three years. LEAPS have proven to be very popular. Although LEAPS are available on index options, most of the trading volume is in LEAPS on individual stocks.

As noted above, FLEX options are available on stocks and indices and permit the investor to specify any exercise price. FLEX options can also have any desired expiration up to five years for index options and three years for options on individual stocks. FLEX options are a response on the part of the options exchanges to the growing over-the-counter market in which options have tailored exercise prices and expirations.

The expiration day of an exchange-traded option is the Saturday following the third Friday of the month. The last day on which the option trades is the third Friday of the month.

Position and Exercise Limits

In the U.S., the Securities and Exchange Commission forces the options exchanges to impose position limits that define the maximum number of options an investor can hold on one side of the market. For example, because they are both bullish strategies, a long call and a short put on the same stock are transactions on the same side of the market. Likewise, a short call and a long put are both bearish strategies and thus are considered to be on the same side of the market. The options exchange publishes the position limit for each stock, which varies from 25,000 to 250,000, depending on the stock's trading volume and number of outstanding shares. Index options do not generally have position limits because they are widely used in large institutional portfolios. Certain traders called market makers have specific exemptions from these position limits.

Exercise limits are similar to position limits. An exercise limit is the maximum number of options that can be exercised on any five consecutive business days by any individual or group of individuals acting together. The figure for the exercise limit is the same as that for the position limit.

The purpose of position and exercise limits is to prevent a single individual or group from having a significant effect on the market. It is not clear, however, that such restrictions are necessary. They do, however, prevent many large investors from using exchange-traded options, and they reduce liquidity. They have probably hurt the options exchanges by forcing institutional investors to take their business to the over-the-counter markets.

OPTIONS EXCHANGES AND TRADING ACTIVITY

Options trading currently exists on several U.S. and many foreign exchanges. Table 2.1 lists the exchanges and their World Wide Web addresses.

Options trading is not confined to these exchanges. Options on futures trade on virtually every futures exchange in the country. We shall defer discussing those contracts and exchanges until Chapter 8.

According to *Futures Industry* magazine data, about 468 million contracts were traded on the Chicago Board Options Exchange (CBOE) and 449 million contracts were traded on the International Securities Exchange (ISE) in 2005. Interestingly, the CBOE has been trading since 1973 and the ISE started trading in May 2000. In 2005 the American Stock Exchange traded about 202 million contracts, the Pacific Stock Exchange traded about 145 million, the Philadelphia Stock Exchange traded about 163 million contracts, and the Boston Options Exchange traded about 78 million contracts. United States option volume totaled over 1.9 billion contracts in 2005. In 1973, 1 million option contracts traded, and 1980 was the first year that more than 100 million options contracts traded. Over 500 million options contracts traded for the first time in 1999.

Although it is difficult to obtain an accurate count of worldwide options volume, *Futures Industry* estimates that global exchange-traded options volume was about 5.9 billion contracts for 2005. As we noted earlier, the over-the-counter options market, whose size is not measured by volume, is estimated by the Bank for International Settlements as $40 trillion notional principal and $1.2 trillion market value at the end of 2005.

Table 2.1 Major Options Exchanges

United States

Chicago Board Options Exchange
Chicago, Illinois
http://www.cboe.com
American Stock Exchange (a division of NASDAQ)
New York, New York
http://www.amex.com
Kansas City Board of Trade
Kansas City, Kansas
www.kcbt.com
Philadelphia Stock Exchange
Philadelphia, Pennsylvania
http://www.phlx.com
International Securities Exchange
New York, New York
http://www.iseoptions.com
Boston Options Exchange
http://www.bostonoptions.com

Canada

Montreal Exchange
Montreal, Quebec
http://www.me.org

South America

Bolsa de Mercadorias & Futuros
Sao Paulo, Brazil
http://www.bmf.com.br
Santiago Stock Exchange
Santiago, Chile
www.bolsadesantiago.com

Europe

Wiener Börse AG
Vienna, Austria
http://www.wbag.at
Euronext
Brussels, Paris, Amsterdam
http://www.euronext.com
Copenhagen Stock Exchange (FUTOP)
Copenhagen, Denmark
http://www.xcse.dk
Helsinki Securities and Derivatives Exchange
Helsinki, Finland
http://www.hex.fi
EUREX
Frankfurt, Germany
http://www.eurexchange.com
EUREX
Zurich, Switzerland
www.eurexchange.com
Italian Stock Exchange
Milan, Italy
http://www.borsaitalia.it
Sibiu Monetary-Financial and Commodities Exchange
Sibiu, Romania
http://www.bmfms.ro
Meff Renta Variable
Madrid, Spain
http://www.bolsasymercados.es/asp/homepage.asp?empresa=8
OM Stockholm Exchange
Stockholm, Sweden
http://www.omgroup.com
London International Financial Futures and Options Exchange (LIFFE)
London, United Kingdom
http://www.euronext.com/home_derivatives/0,4810,1732_6391950,00.html
OM London Exchange
London, United Kingdom
http://www.omgroup.com

Africa

South African Futures Exchange
Republic of South Africa
http://www.safex.co.za

Australia and New Zealand

Australian Stock Exchange
Sydney, Australia
http://www.asx.com.au
Sydney Futures Exchange
Sydney, Australia
http://www.sfe.com.au
New Zealand Futures and Options Exchange
Aukland, New Zealand
http://www.sfe.com.au

Asia

Hong Kong Exchange
Hong Kong, China
http://www.hkex.com.hk
Tel Aviv Stock Exchange
Tel Aviv, Israel
http://www.tase.co.il
Tokyo Stock Exchange
Tokyo, Japan
http://www.tse.or.jp
Korea Futures Exchange
Seoul, Korea
http://www.kofex.com
Korea Stock Exchange
Seoul, Korea
http://www.kse.or.kr
Malaysia Derivatives Exchange
Kuala Lumpur, Malaysia
http://www.mdex.com.my

India

National Stock Exchange of India
Mumbai, India
http://www.nse-india.com

Note: Exchanges on which options on futures trade are covered in Chapter 8.

OPTION TRADERS

In the over-the-counter market, certain institutions, which may be banks or brokerage firms, stand ready to make markets in options. Exchange-listed options, of course, are created on an exchange, which is a legal corporate entity whose members are individuals or firms. Each membership is referred to as a seat. Although the organizational structures of the various exchanges differ somewhat, membership generally entitles one to physically go onto the trading floor and trade options. The following sections discuss the types of traders who operate both on and off the exchange floor. This system of traders is based on the market maker system used by the CBOE and the Pacific Stock Exchange.

Market Maker

An individual who has purchased a seat on the CBOE can apply to be either a market maker or a floor broker. The market maker is responsible for meeting the public's demand for options. When someone from the public wishes to buy (sell) an option and no other member of the public is willing to sell (buy) it, the market maker completes the trade. This type of system ensures that if a private investor wishes to buy a particular option, there will be a seller willing to make an offer, and if one buys an option and later wants to sell it, there will be a buyer available. The market maker offers the public the convenience of immediate execution of trades.

The market maker is essentially an entrepreneur. To survive, the market maker must profit by buying at one price and selling at a higher price. One way this is done is by quoting a bid price and an ask price. The bid price is the maximum price the market maker will pay for the option. The ask price is the minimum price the market maker will accept for the option. The ask price is set higher than the bid price. The difference between the ask and bid price is called the bid-ask spread.

The bid-ask spread is a significant transaction cost for those who must trade with a market maker. To the market maker, however, it represents the reward for the willingness to buy when the public is selling and sell when the public is buying. Bid-ask spreads are discussed further in the section on transaction costs.

Market makers use a variety of techniques to trade options intelligently and profitably. Many look at fundamentals, such as interest rates, economic conditions, and company performance. Others rely on technical analysis, which purports to find signals of the direction of future stock prices in the behavior of past stock prices. Still others rely simply on intuition and experience. In addition, market makers tend to employ different trading styles. Some are scalpers, who try to buy at the bid and sell at the ask before the price moves downward or after the price moves just slightly upward. Scalpers seldom hold positions for more than a few minutes. In contrast, position traders have somewhat longer holding periods. Many option traders, including some scalpers and position traders, are also spreaders, who buy one option and sell another in the hope of earning small profits at low risk. Option spreading strategies are covered in more detail in Chapter 7.

Floor Broker

The floor broker is another type of trader on the exchange. The floor broker executes trades for members of the public. If someone wishes to buy or sell an option, that individual must first establish an account with a brokerage firm. That firm must either employ a floor broker or have an arrangement whereby it contracts with either an independent floor broker or a floor broker of a competing firm.

The floor broker executes orders for nonmembers and earns either a flat salary or a commission on each order executed. The floor broker generally need not be concerned about whether the price is expected to go up or down; however, a good broker will work diligently to obtain the best price for the customer.

The CBOE also has a number of Designated Primary Market Makers, or DPMs, who are allowed to be both market makers and brokers. This system is somewhat controversial because the ability to be both a market maker, who executes trades for her own account, and a broker, who executes trades for others, can lead to abuse. With this practice, known as dual trading, a market maker could take advantageous positions for her own account prior to taking a customer's order to the floor. This would allow the market maker to potentially profit because of knowledge of the customer's order. On the other hand, dual trading also increases liquidity.

Order Book Official

A third type of trader at the CBOE is the order book official (OBO) or board broker, an employee of the exchange. To see how an OBO works, suppose you place a *limit order*—an order specifying a maximum price to be paid on a purchase or a minimum acceptable price on a sale—to buy a call option at a maximum price of $3. The floor broker handling your order determines that the best quote offered by a market maker is 2.75 bid and 3.25 ask. This means that the lowest price at which a market maker will sell the call is 3.25. If your floor broker has other orders to execute, the OBO takes your limit order and enters it into the computer along with all the other public limit orders. The market makers are informed of the best public limit orders. If conditions change such that at least one market maker is willing to quote an ask price of $3 or lower, the OBO will execute your limit order.

Public limit orders are always executed before market maker orders; however, the market makers, being aware of the best public limit orders, know the maximum and minimum prices at which they can trade. For example, if your limit order to buy at 3 is the highest bid and the market maker is quoting an ask price of 3.10, the market maker chooses between accepting your bid and selling the call at 3 or holding out for an offer of 3.05 or higher. If no one bids 3.05 within a reasonable time period, the market maker might choose to take your bid of 3.

The options exchanges also have brought the benefits of modern technology to their order-processing operations, using a variety of electronic means for accelerating the rate at which orders are filled. For example, most CBOE traders now use handheld terminals instead of decks of handwritten cards and pieces of paper to submit and keep track of their trades.

Other Option Trading Systems

The CBOE and the Pacific Stock Exchange use the system of competing market makers. The American and Philadelphia Stock Exchanges use a slightly different system. Here an individual called a specialist is responsible for making bids and offers on options. The specialist maintains and attempts to fill public limit orders but does not disclose them to others. In addition to the specialist are individuals called registered option traders (ROTs), who buy and sell options for themselves or act as brokers for others. Unlike the CBOE market makers, ROTs are not obligated to make a market in the options; market making is the specialist's task.

Most U.S. exchanges and many exchanges elsewhere use pit trading, the form of trading we have described here. Electronic trading systems do away with the pit and allow trading from electronic terminals, which can be placed anywhere, including offices and even homes. There are a number of different electronic trading systems, but, for the most part, they permit bids and offers to be entered into a computer that collects the information and makes it available to participants, who can then execute transactions with the stroke of a key. The International Securities Exchange, located in New York, trades options electronically on many actively traded stocks. In addition, many foreign exchanges are fully automated, including EUREX, one of the world's largest exchanges.

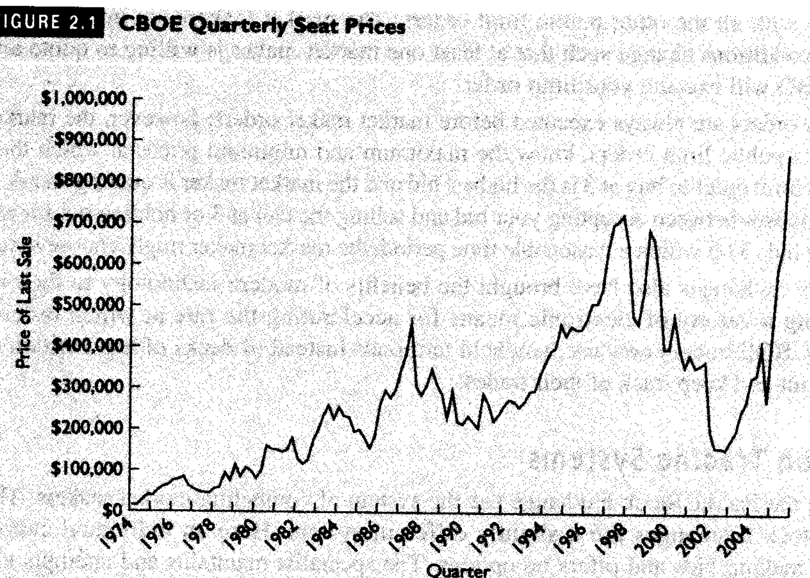
Off-Floor Option Traders

The financial world consists of a vast number of institutions of all sizes, many of which participate in options trading. Some of these institutions are brokerage firms that execute orders for the public. Most brokerage firms employ individuals responsible for recommending profitable option trades to their brokers. Many, however, have specialized option-trading departments that search for mispriced options, trade in them, and in so doing contribute to their firms' profitability. Many large institutional investors, such as pension funds, trusts, and mutual funds, also trade options. In most cases, these types of investors write options on the stocks held in their portfolios. A growing contingent of foreign institutions also trade options. In addition to the large institutional investors, there are, of course, numerous individuals—some wealthy and some not—who trade options. Among those who trade options themselves are many wealthy people who turn over their financial affairs to specialized managers and thus participate in the options market without being personally involved. The dollar amounts required for trading exchange-listed options are so small that virtually anyone can afford to participate.

Cost and Profitability of Exchange Membership

An individual who decides to purchase a seat on an exchange that has options trading can take one of several routes. The most obvious is to purchase a seat from an existing member. Figure 2.1 shows the history of the price of a seat on the CBOE.

FIGURE 2.1 CBOE Quarterly Seat Prices



At a price near the end of 2005 of around $875,000, the cost of obtaining membership would seem prohibitive to most individuals. There are, however, other ways to gain membership. Some seat owners lease their seats to others; rental rates run about 0.50 to 0.75 percent of the seat's price per month. Also, some members allow "trainees" to trade with their seats and charge them a percentage of the trading profits.

These prices are not the full cost of trading. They provide only access to the floor. In addition there are various fees, including $5,400 annual dues. Furthermore, capital must be available to cover losses and a member must arrange for a firm to guarantee his or her creditworthiness. The minimum amount of capital required is about $50,000. Members must also undergo training and pass an examination verifying that they know the rules and procedures, and they must agree to conform to all exchange and SEC regulations. There are additional start-up costs and monthly expenses.

It is difficult to determine how profitable option memberships are. An SEC study in 1978 showed that the average market maker earned a respectable but not unusually large amount of money. That study is outdated, however, because the options markets have changed much since that time, particularly after the crash of 1987. Anecdotal evidence suggests that options market making is a relatively high-risk profession characterized by extreme pressure. The temptation to take undue risks in the hope of making large profits is quite great. Many new market makers end up out of the business rather quickly. Those who are successful generally have pursued conservative strategies. Due to the stress, the typical market maker is a young person in his 20s or early 30s.

MECHANICS OF TRADING

Placing an Opening Order

An individual who wants to trade options must first open an account with a brokerage firm. The individual then instructs the broker to buy or sell a particular option. The broker sends the order to the firm's floor broker on the exchange on which the option trades. All orders must be executed during the normal trading hours, which vary by product from 8:30 A.M. to either 3:02 P.M. or 3:15 P.M. Central time. Trading is performed within the trading pit designated for the particular option. The trading pit is a multilevel, octagonally shaped area within which the market makers and floor brokers stand.

An investor can place several types of orders. A market order instructs the floor broker to obtain the best price. A limit order, as indicated earlier, specifies a maximum price to pay if buying or a minimum price to accept if selling. Limit orders can be either good-till-canceled or day orders. A good-till-canceled order remains in effect until canceled. A day order stays in effect for the remainder of the day. Finally, an investor holding a particular option might place a stop order at a price lower than the current price. If the market price falls to the specified price, the broker is instructed to sell the option at the best available price. There are a number of other types of orders designed to handle different contingencies.

In addition to specifying the option the investor wishes to buy or sell, the order must indicate the number of contracts desired. The order might be a request to purchase ten contracts at the best possible price. The market maker's quote, however, need apply to only one contract. Therefore, if multiple contracts are needed, the market maker may offer a less favorable price. In that case, the order might be only partially filled. To avoid a partial fill, the investor can place an all or none or an all or none, same price order. An all or none order allows the broker to fill part of the order at one price and part at another. An all or none, same price order requires the broker to either fill the whole order at the same price or not fill the order at all.

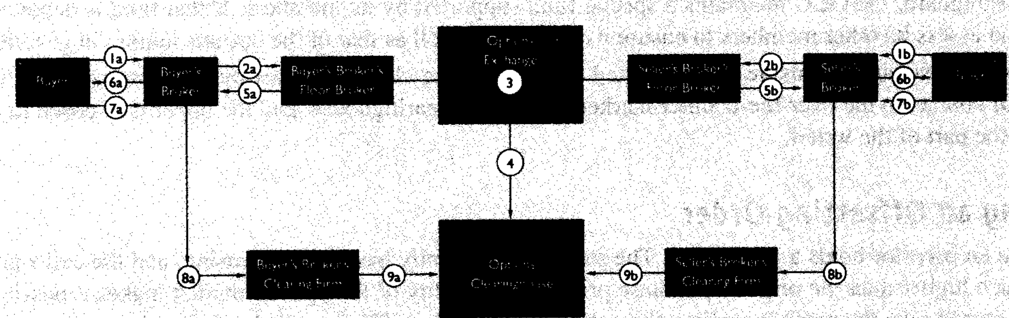
Role of the Clearinghouse

After the trade is consummated, the clearinghouse enters the process. The clearinghouse, formally known as the Options Clearing Corporation (OCC), is an independent corporation that guarantees the writer's performance. The OCC is the intermediary in each transaction. A buyer exercising an option looks not to the writer but to the clearinghouse. The writer of an exercised option makes payment for or delivery of the stock to the clearinghouse.

Each OCC member, known as a clearing firm, has an account with the OCC. Each market maker must clear all trades through a member firm, as must every brokerage firm, although in some cases a brokerage firm is also a clearing firm.

Figure 2.2 illustrates the flow of money and information as an option transaction is consummated and cleared. We shall illustrate how the clearinghouse operates by assuming that you bought the Microsoft

FIGURE 2.2 A Transaction on an Options Exchange



(1a)(1b) Buyer and seller instruct their respective brokers to conduct an options transaction.
(2a)(2b) Buyer's and seller's brokers request that their firms' floor brokers execute the transaction.
(3) Both floor brokers meet in the pit on the floor of the options exchange and agree on a price.
(4) Information on the trade is reported to the clearinghouse.
(5a)(5b) Both floor brokers report the price obtained to the buyer's and seller's brokers.
(6a)(6b) Buyer's and seller's brokers report the price obtained to the buyer and seller.
(7a)(7b) Buyer deposits premium with buyer's broker. Seller deposits margin with seller's broker.
(8a)(8b) Buyer's and seller's brokers deposit premium and margin with their clearing firms.
(9a)(9b) Buyer's and seller's brokers' clearing firms deposit premium and margin with clearinghouse.

Note: Either buyer or seller (or both) could be a floor trader, eliminating the broker and floor trader.

September 27.5 call options we described earlier. You contacted your broker, who, through either his firm's floor broker or an independent floor broker, found a seller. You bought ten contracts at a price of $0.125 per option, which totals $125. The seller, whose identity you do not know, has an account with another brokerage firm. Your brokerage firm clears its trades through XYZ Trading Company, a clearing firm that is a member of the OCC. The seller's broker clears through ABC Options, another member of the OCC. You pay your broker the $1,650, and your broker pays XYZ Trading Company. XYZ pools the transactions of all its customers and, through a predetermined formula, deposits a sum of money with the OCC.

Let us also assume that the seller does not already own the stock, so she will have to deposit some additional money, called margin, with ABC. The amount of margin required is discussed in Appendix 2.A; for now, however, just assume that the amount is 20 percent of the value of the stock, which comes to ($25.92)(100)(10)(0.2) = $5,184. The seller delivers $5,184 to the broker, who deposits it with ABC Options, which also keeps the $125 premium. ABC, in turn, is required to deposit with the OCC an amount of money determined according to a formula that takes its outstanding contracts into account.

The OCC guarantees the performance of ABC, the seller's clearing firm. Thus, you, the buyer, need not worry about whether the shares will be there if you decide to exercise your option. If the shares are not delivered by the seller, the OCC will look to ABC who will look to the seller's brokerage firm, which will look to the seller's personal broker, who will look to the seller for payment or delivery of the shares.

The total number of option contracts outstanding at any given time is called the open interest. The open interest figure indicates the number of closing transactions that might be made before the option expires. In May 2006, open interest in options cleared through the Options Clearing Corporation was about 203 million contracts. About 92 percent of this is in options on individual stocks. Each contract covers 100 shares. Assuming an average stock, priced at about $50, the underlying asset value is about $934 billion. Eight percent of option open interest is index options and less than one percent is currency options.

The OCC thus fulfills the important responsibility of guaranteeing option writers' obligations will be fulfilled. A call buyer need not examine the writer's credit; in fact, in the case of individuals and firms off the floor, the buyers do not even know the writers' identities.

Because the member clearing firms assume some risk, the OCC imposes minimum capital requirements on them. The OCC has a claim on their securities and margin deposits in the event of their default. As a further safeguard, the OCC maintains a special fund supported by its members. If that fund is depleted, the OCC can assess its other members to ensure its survival as well as that of the options market in general.

Options exchanges outside of the United States also use clearinghouses that operate similarly to the OCC. Of course, in the over-the-counter market there is no clearinghouse, and the buyer is exposed to credit risk on the part of the writer.

Placing an Offsetting Order

Suppose an investor holds a call option. The stock price recently has been increasing, and the call's price is now much higher than the original purchase price. The liquidity of the options market makes it possible for the investor to take the profit by selling the option in the market. This is called an offsetting order or simply an offset. The order is executed in the same manner as an opening order. Continuing with our example in which you bought ten contracts of the Microsoft September 27.5 calls, suppose the price of the calls is now $0.50. You instruct your broker to sell the calls. Your broker orders his firm's floor broker to sell the calls. The floor broker finds a buyer who agrees to the price of $0.50. The buyer pays $500 to her broker, who passes the funds through to the company's clearing firm, which passes the funds through to the OCC. The OCC then credits the $500 to the account of your broker's clearing firm, XYZ Trading Company, which credits your broker's firm, which in turn credits your account. You now have $500 and no outstanding position in this option. The individual who bought these options from you may have been offsetting a previously established short position in the calls or may be establishing a new, long position in them. About half of all opening stock option transactions are closed in this manner. In some cases, however, option traders may wish to exercise the option, which we discuss in the next section.

In the over-the-counter markets, there is no facility for selling back an option previously bought or buying back an option previously sold. These contracts are created with the objective of being held to expiration. As circumstances change, however, many holders or writers of over-the-counter options find that they need to reverse their positions. This can be done by simply entering the market and attempting to construct an offsetting position. In other words, if you had previously bought an April 5400 call on London's Financial Times 100 index and now would like to reverse the transaction before expiration, you simply call a dealer and offer to sell the same option. In the over-the-counter market, however, there is not likely to be someone trying to do the exact offsetting trade at the same time. A dealer takes the opposite position, but there are many dealers willing to do the trade. The most important difference between offsetting in the over-the-counter market and offsetting in the exchange-listed market is that in the latter the contracts cancel any obligations of the writer. In the former, both contracts remain on the books, so they are both subject to default risk. In some cases, a reversing trade with the same dealer with whom one did the original contract can be structured as an offset, thereby terminating both contracts.

Exercising an Option

An American option can be exercised on any day up through the expiration date. European options, which have nothing to do with Europe, can be exercised only on the expiration date. Suppose you elect to exercise the Microsoft September 27.5 calls, which, like all options on stocks in the United States, are American options. You notify your brokerage firm, which in turn notifies the clearing firm through which the trade originally was cleared. The clearing firm then places an exercise order with the OCC, which randomly selects a clearing firm through which someone has written the same option. The clearing firm, using a procedure established and made known to its customers in advance, selects someone who has written that option. Procedures such as first-in, first-out or random selection are commonly used. The chosen writer is said to be assigned.

If the option is a call option on an individual stock, the writer must deliver the stock. You then pay the exercise price, which is passed on to the writer. If the option had been a put option on an individual stock, you would have had to deliver the stock. The writer pays the exercise price, which is passed on to you. For either type of option, however, the writer who originally wrote the contract might not be the one who is assigned the exercise.

Because an index represents a portfolio of stocks, exercise of the option ordinarily would require the delivery of the stocks weighted in the same proportions as they are in the index. This would be quite difficult and inconvenient. Instead, an alternative exercise procedure called cash settlement is used. With this method, if an index call option is exercised, the writer pays the buyer the contract multiple times the difference between the index level and the exercise price. For example, assume that you buy one index call option that has a multiple of 100. The index is at 1500, and the exercise price is 1495. If you exercise the option, the assigned writer pays you $100(1500 - 1495) = \$500$ in cash. No stock changes hands. A put option is exercised by the writer paying the buyer the multiple times the difference between the exercise price and the index level.

An order to exercise an index option during the day is executed after the close of trading. The index value *at the end of the day,* rather than the index value when the exercise was ordered, is used to determine the amount of the settlement. Thus, an investor is well advised to wait until the end of the day to order an exercise of an index option. In addition, certain index options are settled based on the index value at the opening of the next day.

For a call option on the expiration day, if you find that the stock price is less than the exercise price or, for a put, that the stock price is greater than the exercise price, you allow the option to expire by doing nothing. When the expiration day passes, the option contract is removed from the records of the clearing firm and the OCC.

About 10 to 20 percent of options on stocks are exercised and about a third expire with no value. In some cases, options that should have been exercised at expiration were not, due to customer ignorance or lack of oversight. Some brokerage firms have a policy of exercising options automatically when doing so is to the customer's advantage. Such a policy is usually stated in the agreement that the customer signs when opening the account. The OCC automatically exercises options that are at least slightly in-the-money at expiration.

OPTION PRICE QUOTATIONS

Option prices are available daily in *The Wall Street Journal* and in many newspapers in large cities. The print version of *The Wall Street Journal* covers only the most active option classes. Thus, on any given day you will see the options of fewer than 100 stocks. The prices of all options are available on the Journal's Web site, http://www.wsj.com, which is accessible to subscribers of the print edition. The print version of *The Wall Street Journal* provides the price of the last trade and the volume for the day, along with the exercise price, expiration month, and closing stock price. The online version also provides the open interest for each option.

One problem with option prices obtained from newspapers, however, is that by the time the prices are available, they are quite outdated. In addition, the price of an option is not necessarily synchronized with the indicated closing price of the underlying stock. The last transaction of the day for the option need not occur near the same time as the last transaction of the day for the stock. For one, the options market and the market where the stock trades may not even close at the same time. Also, for thinly traded options and/or stocks, the last trade for each can easily occur hours apart.

Another problem with newspaper prices is that they are transaction prices and not bid and ask prices. These are the prices at which the market maker will buy and sell to the public. Suppose that the option bid price is 3.25 and the ask price is 3.30. If the last trade of the day is a public order to buy, the closing price will be 3.30. If the last trade of the day is a public order to sell, the closing price will be 3.25.

As a result of these problems and for numerous other obvious reasons, the best sources of option price information are the Web sites of the exchanges. All of the U.S. exchanges follow a similar format in reporting option prices. Real-time price quotes are typically available on a fee-only basis; however, 15-minute-delayed quotes are available free. In addition, the options exchanges show not only the last sale, but also the current bid and ask price. The latest stock price is also shown, but, again, there is no guarantee that the indicated stock price is synchronized with the option quotes. That is, while the option bid and ask prices may be accurate and current (subject to the 15-minute delay), the indicated stock price will simply be the last trade, which may have occurred hours earlier. If one uses the stock price to help determine the value of an option, which is the focus of Chapters 4 and 5, the stock price may be stale and misleading. Nonetheless, having the bid and ask prices are better than having only the last trade price. If the stock bid and ask prices and the time of the last stock trade are needed, they can usually be obtained from sites such as http://finance.yahoo.com or from a brokerage service Web site.

Both *The Wall Street Journal* and the exchange Web sites show volume and open interest for options on U.S. exchanges and the overall total for each exchange.

See the "Derivative Tools: Concepts, Applications, and Extensions, Reading Option Price Quotations" section for information on how to access and interpret this information.

TYPES OF OPTIONS

Two popular types of options are stock options and index options. We shall touch only briefly on these here, as they are the major focus of discussions through Chapter 7. We will also briefly review currency options, interest rate options, real options, as well as many other types of options.

Stock Options

Options on individual stocks are sometimes called stock options or equity options. These options are available on several thousand individual stocks, though trading volume may be low for options on certain stocks. In addition, certain options on virtually every stock, such as options with long expirations and options that are either deep-in-the-money or deep-out-of-the-money, have very low trading volume. In the U.S., the options of many stocks trade on more than one exchange.

DERIVATIVES TOOLS

Concepts, Applications, and Extensions
Reading Option Price Quotations

The following example illustrates the price quotations for Microsoft® options during the trading day of September 2, 2005.

Option price quotations can be obtained from the Web sites of the options exchanges. When accessing an option quote, you are typically given a choice of whether to enter the stock ticker symbol or the option symbol. Stock ticker symbols can found on the exchanges' Web sites, or on other Web sites such as http://finance.yahoo. com, or in The *Wall Street Journal's* stock quotation pages.

The ticker symbols of New York Stock Exchange-listed stocks are one, two, or three letters. The ticker symbols of NASDAQ-listed stocks are four letters. Microsoft is a NASDAQ stock, and its symbol is MSFT. To look up option quotations on Microsoft, go to the Web site http://www.cboe.com. From the menu, choose "Quotes." Then select the free "Delayed Quotes" link. From there, you can enter the stock ticker symbol, along with choices regarding whether you wish to see all option quotes, regardless of the exchange on which the option is traded, or certain other restrictions that can make the amount of data returned to you either very large or more limited.

On September 2, 2005 at 2:40 p.m. Eastern time, the quote below was obtained for two Microsoft options.

These are the options expiring in September of 2005 with an exercise price of 27.50. For calls, the last trade was at $0.20, which was down $0.05. The current bid is $0.15 and the current ask is $0.20. So far that day, 146 contracts had traded. Over the life of the contract, 113,106 contracts had been opened and not yet closed or exercised. Volume and open interest statistics include all exchanges. Similar information is provided for the puts.

Calls	Last Sale	Net	Bid	Ask	Vol	Open Int	Puts	Last Sale	Net	Bid	Ask	Vol	Open Int
05 Sep 27.50 (MSQIY-E)	0.20	°0.05	0.15	0.20	146	113,106	05 Sep 27.50 (MSQUY-E)	0.45	C0.05	0.45	0.50	452	27,710

Notice in the boxes labeled "Calls" and "Puts" that there is a symbol in parentheses: MSQIY-E for calls and MSQUY-E for puts. These symbols are referred to as option codes and can be used instead of the stock ticker symbol to look up the prices of particular options. The first three letters of the option symbol is a three-letter root. The options of a given stock, however, can have several different roots. For example, Microsoft's root is either MQF or MSQ. Appended to the root are two other letters, the first of which represents a combination of the expiration month and type of option (call or put) and the second of which is an indication of the exercise price. The first letter after the root is interpreted from the table below.

Note that the code for the Microsoft calls is the letter *I* and the code for the Microsoft puts is the letter *U*. From this table, we see that this letter refers to the September calls and puts.

Interpretation of the exercise price code requires a large table, which can be accessed on the exchanges' Web sites. Notice that Microsoft's code for the calls and puts is Y. Referring to the chart from the CBOE's Web site, we find that *Y* can stand for exercise prices of 27 1/2, 57 1/2, 87 1/2, 117 1/2, or various higher prices with one-half point fractions. Knowing that Microsoft's stock is trading at around $27, you would know that this exercise price is 27 1/2. Different codes are used for index options, however, because stock index levels can be on a completely different order of magnitude.

The final letter in the option symbol refers to the exchange on which the option trades. E stands for the CBOE, A stands for the American Stock Exchange, X is the Philadelphia Exchange, P is the Pacific Exchange, and 8 is the International Securities Exchange.

Thus, if you wished to obtain a quote only for the September 27 1/2 call trading on the CBOE, you would enter MSQ IY-E (the space is required). MSQ indicates Microsoft, I indicates the September calls, Y indicates an exercise price of 27 1/2 and E indicates the Chicago Board Options Exchange. If you wanted the September 30 calls, you would enter MSQ IF-E. Thus, you have to know the three-letter code. In general, it is probably just as easy to enter the Microsoft ticker symbol and obtain the quotes of numerous options on Microsoft.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Calls	A	B	C	D	E	F	G	H	I	J	K	L
Puts	M	N	O	P	Q	R	S	T	U	V	W	X

Index Options

A stock index is a measure of the overall value of a designated group of stocks. As in any index, it is a relative measure, capturing value relative to a previous value. For index options, we interpret the quoted level as the market value of the stocks relative to a base level value, typically created many years ago when the index was initiated. The first option on a stock index was launched at the CBOE in 1983 and index options have been highly successful ever since. Since that time, a number of index options have been added and many, but not all, have been very actively traded. The option on the NASDAQ 100 stock index is typically the most active of all options in the United States. In most countries, there is an index option trading in which the underlying is a stock index representing the most actively traded stocks of that country's market. These are also usually among the most actively traded of all options in a given country.

Index options are available on broad-based indices, such as the S&P 500 and NASDAQ 100, and also on more narrowly defined indices. The widely followed Dow Jones Industrial Average consists of only 30 stocks, but its index options are very actively traded. In addition, there are index options on various industry indices (e.g., technology, telecommunications, oil) and various market sectors (e.g., large-capitalization stocks, medium-capitalization stocks, and small-capitalization stocks).

Index options have been popular for two reasons. First, they are nearly always designed to be cash settled at expiration, which enables investors to trade options without having to take or make delivery of stock. Cash settlement has, however, been both a blessing and a curse. Many institutional investors hold large portfolios of stocks that purport to replicate the index. They then write call options or sell futures against these portfolios. At other times, these investors sell short the stocks and buy call options or futures. When the options approach the expiration date, the institutional investors, not wanting to hold the stocks after the options expire, attempt to unwind their stock positions. Holders of stock sell the stock, and short sellers buy it back. As a result, many large stock transactions are made near expiration and are accompanied by increased volatility. The cash settlement feature of index options and futures has been blamed for this volatility, although it is not clear that it is truly the cause.

The second reason for the popularity of index options is the fact that they are options on the market as a whole. Because there are so many stocks on which options trade, few investors have the time to screen that many opportunities. Many prefer to analyze the market as a whole and use index options (and futures) to act on their forecasts.

Currency Options

A foreign exchange rate is the price at which a party can exchange one currency for another currency. For example, if the exchange rate for euros (€) against dollars is $0.98 per €, one can give up $0.98 and receive

€1. Alternatively, the exchange rate can be inverted to obtain €1/$0.98 = €1.0204. Thus, one could give up €1.0204 and receive $1. A currency can be viewed as an asset, much like a stock or bond. Like a stock or a bond price, the currency price, which is the exchange rate, fluctuates in a market. One can buy the currency and deposit it into a foreign bank, during which time it will accrue interest at the foreign interest rate. The interest can be viewed like the interest on a bond or the dividends on a stock.

Because many firms and investors have exposure to exchange rate risk, options and other currency derivatives are widely used. A currency option contract specifies an exercise price, expressed in terms of an exchange rate, an expiration, the identity of the underlying currency, the size of the contract, and various other specifications similar to those of stock and index options.

Exchange-listed currency options trade on the Philadelphia Stock Exchange, though trading activity is very low. The more active market is the over-the-counter currency options market. The Bank for International Settlements estimates that the notional principal of currency options at year-end 2005 was about $7.2 trillion, with a market value of about $139 billion.

Other Types of Options

The options exchanges have experimented with a number of different types of options, including options on bonds, though these have attracted little interest. Options on bonds and related options called *interest rate options* are, however, extremely popular in the over-the-counter markets. We shall explore these in Chapter 13. As previously noted, there are also options on futures, a topic which we shall defer discussing until Chapter 8, when we cover futures.

Many financial institutions now offer securities on which they pay interest based on a minimum value plus the performance of the stock market above a specific level. This is like a call option plus a bond. Other innovative options include some that pay off based on how the price of a commodity, such as oil, performs. It is possible to trade an option that pays off based on which stock index, say the S&P 500 or London's Financial Times 100, performs better. There are also options that expire if the stock price falls to a certain level; options that are based on the average, maximum, or minimum stock price during the life of the option; and an option that allows you to decide, after buying it but before it expires, whether to make it a put or a call. These options, often called exotics, are but a sampling of the tremendous number of innovations that have developed in the over-the-counter markets in recent years. We shall take a look at some of these kinds of instruments in Chapter 14.

A number of other common instruments are practically identical to options. For example, many corporations issue warrants, which are long-term options to buy the companies' stock. Warrants are often issued in conjunction with a public offering of debt or equity. Many corporations issue convertible bonds, which allow the holder to convert the bond into a certain number of shares of stock. The right to convert is itself a call option. Callable bonds, which give the issuing firm the right to repay the bonds early, contain an option-like component. Stock itself is equivalent to a call option on the firm's assets written by the bondholders with an exercise price equal to the amount due on the debt. Executive stock options, which are call options written by a corporation and given to its executives, are used extensively as compensation and to give executives a strong incentive to engage in activities that maximize shareholder wealth.

Real Options

Real options are not options on real estate, nor does the name *real options* suggest that other options are somehow *not real*. Real options are options that are commonly found in corporate investment decisions, which are themselves often referred to as real investments.

Consider for example a firm that builds a new plant. Once the plant is built and operating, the company usually has the opportunity to expand it, contract it, temporarily shut it down, terminate it, or sell it to another firm. These options are similar to ordinary options. They allow the owner, which is the firm, to decide at a future date whether to "exercise" the option, which can entail paying an additional sum of money and receiving something of greater value (as in a call)—or receiving a sum of money and giving up something of lesser value (as in a put). These examples are among the simplest of real options. Let us now consider a more complex real option.

Consider a pharmaceutical company that is considering whether or not to invest in research and development (R&D), clinical trials, government approval, and subsequent production and marketing of a new drug. Suppose that the preliminary stages have the following times and costs:

Research and development	3 years	$100 million
Clinical trials	5 years	$200 million
Government approval	2 years	$20 million

If the firm elects to invest $100 million in three years of R&D and the R&D successfully results in developing a drug, the firm will subsequently face another decision: whether to invest $200 million in five years of clinical trials. At the end of five years, if the clinical trials are successful, the firm must then decide whether to invest two years and $20 million in obtaining governmental approval to manufacture and sell the drug. If governmental approval is obtained, let us assume that the firm would then need to decide whether to invest $500 million in manufacturing and marketing the drug.

Traditional capital investment analysis, such as net present value, is unable to adequately capture the value of the flexibility. Capital investment decisions are often made sequentially as additional information is available. For example, the firm will not decide whether to begin clinical trials until the results of the R&D period are in. It will not decide whether to seek governmental approval for the drug until the results of the clinical trials are in.

These decisions can be viewed as options. They are the choices the firm has to exercise, by investing these sums of money, or not, at a future date, conditional on the information available at those times. While the example given here is a relatively complex series of interrelated options, it should be easy to see that the company has a sequence of call options.

Real options have many of the characteristics of ordinary options. Yet in many ways, they are quite different. Real options have rights that clearly have value, but oftentimes the expirations and exercise prices of real options are not always clearly known. For example, does the pharmaceutical company know today exactly how long governmental approval will require or what it will cost? Real options are often based on vaguely specified underlying estimates of costs or benefits that either do not trade in a liquid market or do not trade at all. In the example mentioned earlier in the paragraph, the underlying includes the market value of the drug, which cannot be accurately assessed until it is marketed.

In spite of these and many other limitations, the use of option models for estimating the value of real options is a growing interest among academics and practitioners. While we cannot easily identify real options, we know that they are there and that they have value. You will have an opportunity to study them when you study the financial management of corporations. What you will learn in this book will be extremely helpful for understanding real options.

TRANSACTION COSTS IN OPTION TRADING

Option trading entails certain transaction costs. The costs depend on whether the trader is a member of the exchange, a nonmember institutional investor, or a member of the public who is trading through a broker. This section discusses the different types of transaction costs.

Floor Trading and Clearing Fees

Floor trading and clearing fees are the minimum charges assessed by the exchange, the clearing corporation, and the clearing firms for handling a transaction. For trades that go through a broker, these fees are included in the broker's commission, as is discussed in the next section. For market makers, the fees are collected by the market maker's clearing firm.

The clearing firm enters into a contractual arrangement with a market maker to clear trades for a fee usually stated on a per contract basis.

The cost to a market maker is usually less than $1.00 per transaction, though this can vary based on the volume of trades. In addition, market makers incur a number of fixed costs.

For over-the-counter options, these types of costs are not incurred; however, comparable, if not higher, costs are associated with a firm's trading operation and with processing these customized transactions.

Commissions

One of the main advantages of owning a seat on an exchange is that it lets one avoid paying commissions on each trade. The market maker pays indirectly via the opportunity cost associated with the funds tied up in the seat price, the labor involved in trading, and forgoing the earnings that would be realized in another line of work. The savings in commissions, however, are quite substantial.

Discount brokers offer the lowest commission rates, but frequent or large trades are sometimes necessary for taking advantage of discount brokers' prices. A discount brokerage firm does not provide the advice and research that is available from full-service brokers charging higher commission rates. One should not, however, automatically assume that a full-service broker is more costly.

Options commissions are fairly simple, based on a fixed minimum and a per contract charge. Internet rates of about $20 plus $1.25 per contract are available from major discount brokers. Some brokers may set charges based on the total dollar size of the transaction, and some offer discounts to more active customers.

When exercising a stock option, the investor must pay the commission for buying or selling the stock. (Stock commissions are discussed in a later section.) If an investor exercises a cash settlement option, the transaction entails only a bookkeeping entry. Some brokerage firms do not charge for exercising a cash settlement option. When any type of option expires unexercised, there is normally no commission.

For over-the-counter options, commissions are not generally incurred, because the option buyer or seller usually trades directly with the opposite party.

Bid-Ask Spread

The market maker's spread is a significant transaction cost. Suppose the market maker is quoting a bid price of 3 and an ask price of 3.25 on a call. An investor who buys the call immediately incurs a "cost" of the bid-ask spread, or a 0.25 point; that is, if the investor immediately sold the call, it would fetch only $3, the bid price, and the investor would immediately incur a 0.25 point, or $25, loss. This does not, however, mean that the investor cannot make a profit. The call price may well increase before the option is sold, but if the spread is constant, the bid price must increase by at least the amount of the bid-ask spread before a profit can be made.

The bid-ask spread is the cost of immediacy—the assurance that market makers are willing and able to buy and sell the options on demand. The cost is not explicitly observed, and the investor will not see it on the monthly statement from the broker. It is, however, a real cost and can be quite substantial, amounting to several percent of the option's price.

It may appear that market makers can avoid the bid-ask spread transaction cost. This is true in some cases. If, however, the market maker must buy or sell an option, there may be no public orders of the opposite position. In that case, the market maker would have to trade with another market maker and thus would incur the cost of the bid-ask spread.

In the over-the-counter market, the buyer or seller trades directly with the opposite party. In many cases, one of the parties is a financial institution that makes markets in whatever options its clients want. Thus, the client will probably face a bid-ask spread that could be quite significant; but of course, the client is free to shop around.

Other Transaction Costs

Option traders incur several other types of transaction costs. Some of these costs, such as margins and taxes, are discussed in Appendixes 2.A and 2.B, respectively. Most option traders also trade stocks. Thus, the transaction costs of stock trades are a factor in option trading costs.

Stock trading commissions vary widely among brokerage firms; however, 1 percent to 2 percent of the stock's value for a single purchase or sale transaction is a reasonable estimate. Market makers normally obtain more favorable rates from their clearing firms. Also, large institutional investors can usually negotiate volume discounts from their brokers. Internet rates of $10 or less are sometimes seen. Stocks, however, also have bid-ask spreads; ultimately 1 to 2 percent is probably a good estimate of stock trading costs for public investors.

All the transaction costs discussed here are for single transactions. If the option or stock is subsequently purchased or sold in the market, the transaction cost is incurred again.

REGULATION OF OPTIONS MARKETS

In the U.S., the exchange-traded options industry is regulated at several levels. Although federal and state regulations predominate, the industry also regulates itself according to rules and standards established by the exchanges and the Options Clearing Corporation.

The Securities and Exchange Commission (SEC) is the primary regulator of the options market in the U.S. The SEC is a federal regulatory agency established in 1934 to oversee the securities industry, which includes stocks, bonds, options, and mutual funds. The SEC's general purpose is to ensure full disclosure of all pertinent information on publicly offered investments. It has the authority to establish certain rules and procedures and to investigate possible violations of federal securities laws. If the SEC observes a violation it may seek injunctive relief, recommend that the Justice Department press charges, or impose some sanctions itself. For more information on the SEC, see its Web site at http://www.sec.gov.

The exchanges establish rules and procedures that apply to all members as well as to individuals and firms participating in options transactions. Rule violations are punishable by fines and/or suspensions. The Options Clearing Corporation also regulates its members to help ensure that all activities in the options markets are proper and do not pose a risk to the market's viability.

The regulatory authority of an individual state extends to any securities or options trading occurring within that state. States with significant option trading, such as Illinois and New York, actively enforce their own laws on the propriety of transactions conducted therein. Many important issues in the options industry as a whole are sometimes settled in state courts in Illinois and New York.

Other levels of regulation are imposed by the Federal Reserve System, which regulates the extension of margin credit; the Securities Investor Protection Corporation, which provides insurance against the failure of brokerage firms; and the National Association of Securities Dealers (NASD), of which most firms involved in options trading are members. In addition, several regional and national organizations, such as the CFA Institute, indirectly regulate the industry by prescribing ethical standards for their members.

Outside of the United States, the options industry is regulated by federal government regulatory agencies. In most countries options and futures are regulated by the same federal agency. In the United States, these products are regulated by separate agencies, with the Commodity Futures Trading Commission (CFTC), regulating the futures industry.

Many new exchange-traded options products were introduced in the 1980s, including options on stock indices, options on foreign currencies, and options on futures. These products created some confusion as to whether the SEC or the CFTC had regulatory purview. The options on futures instruments caused the greatest confusion, because it is similar to both an option and a futures. In an important step in resolving the matter, the then chairmen of these agencies—John Shad of the SEC and Phillip McBride Johnson of the CFTC—reached an agreement. In what has come to be known as the Johnson-Shad agreement, or CFTC-SEC accord, it was decided that the SEC would regulate options on stocks, stock indices, and foreign currencies, whereas the CFTC would govern options on all futures contracts. Also, a CFTC-regulated contract cannot permit delivery of instruments regulated by the SEC. Although the Johnson-Shad agreement was a milestone in regulatory cooperation, continued disputes between the SEC and the CFTC characterized the regulatory environment of the early 1990s. We shall learn more about these issues when we cover futures.

The primary purpose of the exchange-traded regulatory system is to protect the public. Over the years, many controversial issues have been raised and discussed. In an industry as large as the options industry, some abuses are certain to occur. In recent years, the options industry has been subjected to criticisms that it has manipulated the stock market and abused the public's trust by charging exorbitant prices for options. There is no evidence, however, that any of these charges are true. There have even been a few defaults by writers, but, thanks to the clearinghouse, in no case has any buyer lost money because of a writer's failure to perform. The options industry works hard to maintain the public's trust by operating in an environment of self-regulation. By policing itself and punishing wrong-doers, some of the cost of having federal regulation is offset for taxpayers.

As noted earlier in the chapter, the over-the-counter market is an unregulated market, bound loosely by customs and accepted procedures. The firms that participate, however, are often regulated by the NASD, the Federal Reserve, or the Comptroller of the Currency. Of course, commercial laws always apply. The SEC and the CFTC, however, have no direct regulatory authority over the over-the-counter options market.

Questions and Problems

1. Compare and contrast the exercise procedure for stock options with that for index options. What major advantage does exercising an index option have over exercising a stock option?
2. Contrast the market maker system of the CBOE and Pacific Stock Exchange with the specialist system of the AMEX and Philadelphia Stock Exchange. What advantages and disadvantages do you see in each system?
3. Why are short puts and long calls grouped together when considering position limits?
4. Determine whether each of the following arrangements is an option. If so, decide whether it is a call or a put and identify the premium.
   a. You purchase homeowner's insurance for your house.
   b. You are a high school senior evaluating possible college choices. One school promises that if you enroll, it will guarantee your tuition rate for the next four years.
   c. You enter into a noncancelable, long-term apartment lease.
5. Discuss the limitations of prices obtained from newspapers such as *The Wall Street Journal* and the advantages of quotes obtained from Web sites of the exchanges.
6. Name and briefly describe at least two other instruments that are very similar to options.
7. What adjustments to the contract terms of CBOE options would be made in the following situations?
   a. An option has an exercise price of 60. The company declares a 10 percent stock dividend.
   b. An option has an exercise price of 25. The company declares a two-for-one stock split.
   c. An option has an exercise price of 85. The company declares a four-for-three stock split.
   d. An option has an exercise price of 50. The company declares a cash dividend of $.75.
8. Explain the difference between an American option and a European option. What do they have in common?
9. Explain each of the terms in the following description of an option: AT&T January 65 call.
10. Explain the major difference between the regulation of exchange-traded options and over-the-counter options.
11. Identify and briefly discuss the various types of option transaction costs. How do these costs differ for market makers, floor brokers, and firms trading in the over-the-counter market?
12. Explain how real options are similar to, but different from, ordinary options.
13. Consider the January, February, and March stock option exercise cycles discussed in the chapter. For each of the following dates, indicate which expirations in each cycle would be listed for trading in stock options.

a. February 1
b. July 1
c. December 1

14. Suppose you are an individual investor with an options account at a brokerage firm. You purchase 20 call contracts at a price of $2.25 each. Explain how your premium ends up at the clearinghouse.
15. Compare and contrast the roles of market maker and floor broker. Why do you think an individual cannot generally be both?
16. Explain how the CBOE's order book official (OBO) handles public limit orders.
17. Discuss the three possible ways in which an open option position can be terminated. Is your answer different if the option is created in the over-the-counter market?

Appendix 2.A

Margin Requirements

Margin is the amount of money an individual commits when entering into an investment. The remainder is borrowed from the brokerage firm. The objective of a margin trade is to earn a higher return by virtue of investing less of one's own funds. This advantage is, however, accompanied by increased risk. If the value of the investment does not move sufficiently in the desired direction, the profit from the investment may not be enough to pay off the loan.

Regulation T of the Federal Reserve Act authorizes the Federal Reserve to regulate the extension of credit in the United States. This authority extends to the regulation of margin credit on transactions in stocks and options.

The initial margin is the minimum amount of funds the investor supplies on the day of the transaction. The maintenance margin is the minimum amount of funds required each day thereafter. On any day on which a trade is executed, the initial margin requirement must be met.

The rules presented here apply to public investors. Specialists and market makers have more lenient margin requirements. Clearing firm margins deposited with the OCC are calculated differently based on a netting out of certain identical long and short positions and an additional amount based on the probabilities of customer defaults. In addition, these requirements follow U.S. law and apply only to exchange-listed options.

Margin Requirements on Stock Transactions

The minimum initial margin for stock purchases and short sales is 50 percent. The minimum maintenance margin is 25 percent. Many brokerage firms add an additional 5 percent or more to these requirements.

Margin Requirements on Option Purchases

Options with maturities of nine months or less must be paid for in full. That is, the margin requirement is 100 percent. Options with maturities of greater than nine months can, however, be margined. An investor can borrow up to 25 percent of the cost of these options.

Margin Requirements on the Uncovered Sale of Options

An uncovered call is a transaction in which an investor writes a call on stock not already owned. If the option is exercised, the writer must buy the stock in the market at the current price, which has no upper limit. Thus, the risk is quite high. Many brokerage firms do not allow their customers to write uncovered calls. Those that do usually restrict such trades to wealthy investors who can afford large losses; yet even these must meet the minimum margin requirements.

For an uncovered call, the investor must deposit the premium plus 20 percent of the stock's value. If the call is out-of-the-money, the requirement is reduced by the amount by which the call is out-of-the-money. The margin must be at least the option market value plus 10 percent of the value of the stock. Consider an investor who writes one call contract at an exercise price of \$30 on a stock priced at \$33 for a premium of \$4.50. The required margin is 0.2(\$3,300) + \$450 = \$1,110. If the stock is priced at \$28 and the call is at \$0.50, the margin is 0.2(\$2,800) + \$50 − \$200 = \$410. These amounts are greater than 10 percent of the option value plus the value of the stock. The amount by which the call is out-of-the-money reduces the required margin.

The same rules apply for puts except that the 10 percent rule is applied to the aggregate exercise price. If a put is written at an exercise price of \$30 when the stock price is \$33 and the put price is \$2.375, the required margin is 0.2(\$3,300) + \$237.50 − \$300 = \$597.50. If the stock price is \$29 and the put price is \$3.25, the margin is 0.2(\$2,900) + \$325 = \$905. These amounts are greater than 10 percent of the aggregate exercise price of \$3,000 plus the option value.

Broad-based index options are somewhat less volatile than options on individual stocks and, appropriately, have lower margin requirements. The required margin is 15 percent, instead of 20 percent, of the stock's value.

Margin Requirements on Covered Calls

A covered call is a transaction in which an investor writes an option against a stock already owned. If the option's exercise price is at least equal to the stock price, the investor need not deposit any additional margin beyond that required on the stock. Also, the premium on the option can be used to reduce the margin required on the stock. If, however, the exercise price is less than the stock price, the maximum amount the investor can borrow on the stock is based on the call's exercise price rather than on the stock price. For example, if the stock price is \$40 and the exercise price is \$35, the investor can borrow only 0.5(\$3,500) = \$1,750, not 0.5(\$4,000) = \$2,000, on the purchase of the stock.

Although it is possible to hold a portfolio of stocks that is identical to the index, current margin requirements do not recognize covered index call writing. Therefore, all short positions in index options must be margined according to the rules that apply to uncovered writing.

There are exceptions to many of these rules, particularly when spreads, straddles, and more complex option transactions are used. In these cases, the margin requirements often are complicated. Investors should consult a broker or some of the trade-oriented publications for additional information on margin requirements for the more complex transactions.

Questions and Problems

1. Suppose that a stock is currently priced at \$50. The margin requirement is 20 percent on uncovered calls and 50 percent on stocks. Calculate the required margin, in dollars, for each of the following trades:
   a. Write 10 call contracts with an exercise price of 45 and a premium of 7.
   b. Write 10 call contracts with an exercise price of 55 and a premium of 3.
   c. Write 10 put contracts with an exercise price of 45 and a premium of 3.

d. Write 10 put contracts with an exercise price of 55 and a premium of 7.
e. Buy 1,000 shares of stock and write 10 call contracts with an exercise price of 45 and a premium of 7.
f. Buy 20 put contracts with an exercise price of 50 and a premium of 5.

Appendix 2.B

Taxation of Option Transactions

Determining the applicable taxation of many investment transactions is a complex process that requires the advice of highly trained specialists. The rules covered in this appendix outline most of the basic option transactions. There are many exceptions and loopholes, and the laws change frequently. In all cases, one should secure competent professional advice. Of course, these rules are based on U.S. tax law.

The ordinary income of most individuals is taxed at either 15, 28, 31, 36, or 39.6 percent. Long-term capital gains, defined as profits from positions held more than 12 months, are taxed at a maximum of 20 percent. Although there are some exceptions noted herein, most exchange-listed options profits are short-term.

In these examples, we ignore brokerage commissions; however, they are deductible. The commission on the purchase of the asset is added to the purchase price; the commission on its sale is subtracted from the sale price.

The deductibility of any losses is determined by offsetting them against other investment gains. If the losses exceed the gains, any excess up to $3,000 per year is deductible against the investor's ordinary income.

These rules apply to individual investors. Corporations and other institutions trading options, especially over-the-counter options, may be subject to different tax rules and interpretations.

For the following examples, we shall assume the investor is in the 28 percent tax bracket.

Taxation of Long Call Transactions

If an investor purchases a call and sells it back at a later date at a higher price, there is a capital gain, which is either long- or short-term, usually short-term. Thus, it is taxed at the ordinary income rate. If the investor sells the call at a loss or allows it to expire, the loss is deductible as previously described.

Consider an investor who purchases a call at $3.50 on a stock priced at $36 with an exercise price of $35. Less than 12 months later, but before expiration, the investor sells the call at $4.75. The profit of $1.25 is taxed at 28 percent for a tax of $1.25(0.28) = $0.35. If the call were sold at a loss, the loss would be deductible as described earlier.

If the investor exercised the call, the call price plus the exercise price would be treated as the stock's purchase price and subsequently used to determine the taxable gain on the stock. For example, if the above call were exercised when the stock price was $38, the purchase price of the stock would be considered as the $35 exercise price plus the $3.50 call price for a total of $38.50. If the investor later sold the stock for $40, the taxable gain would be $1.50 and the tax $1.50(0.28) = $0.42. A tax liability or deduction arises only when the stock is subsequently sold.

Taxation of Short Call Transactions

If an investor sells a call and later repurchases it or allows it to expire, any profit is a short-term capital gain and taxed at the ordinary income rate. If the call is exercised, the exercise price plus the call price is treated as the price at which the stock is sold. Then the difference between the price at which the stock is sold and the price at which it is purchased is the taxable gain.

For example, if the above investor writes the call at a price of $5.25 and subsequently buys it at $3.50 before expiration, the profit of $1.75 will be taxed at 28 percent for a tax of $1.75(0.28) = $0.49. A loss would be deductible as previously described. If the call were exercised, the writer would deliver the stock. Suppose the stock price were $38 and the writer did not already own the stock. Then the writer would purchase the stock for $38 and sell it to the buyer for $35. The sum of the exercise price plus the premium, $35 + $5.25 = $40.25, would be treated as the stock's sale price. The taxable gain to the writer would be $2.25 and the tax would be $2.25(0.28) = $0.63. Had the writer purchased the stock at an earlier date at $30, the taxable gain would be $10.25 and the tax would be $10.25(0.28) = $2.87.

Taxation of Long Put Transactions

If an investor purchases a put and sells it or allows it to expire less than 12 months later, the profit is a short-term capital gain and is taxed at the ordinary income rate. If the put is exercised, the exercise price minus the premium is treated as the stock's sale price. Then the profit from the stock is taxed at the ordinary income rate.

Consider an investor in the 28 percent tax bracket who purchases a put at $3 on a stock priced at $52 with an exercise price of $50. Later the investor sells the put for $4.25. The gain of $1.25 is taxed at 28 percent for a tax of $1.25(0.28) = $0.35. If the put is sold at a loss, the loss is deductible as previously described.

Suppose the investor exercises the put when the stock price is $46. The law would treat this as the sale of stock at $50 less the premium of $3 for a net gain of $47. If the investor purchases the stock at $46 and exercises the put, the taxable gain is $1 and the tax is $1(0.28) = $0.28. If the investor had previously purchased the stock at $40, the taxable gain would be $7 and the tax would be $7(0.28) = $1.96. Had the investor purchased the stock at a price higher than $47, the loss would be deductible as described earlier.

Another possibility is that the investor uses the exercise of the put to sell short the stock. Recall from Chapter 1 that a short sale occurs when an investor borrows stock from the broker and sells it. If the stock price falls, the investor can buy back the stock at a lower price, repay the broker the shares, and capture a profit. A short sale is made in order to profit in a falling market. In our example, the stock is sold short at $47. When the investor later repurchases the stock, any gain on the stock is taxable or any loss is deductible.

Taxation of Short Put Transactions

If an investor writes a put and subsequently buys it back before expiration, any gain is considered a short-term capital gain and is taxed at the ordinary income rate and any loss is deductible. If the put is exercised, the put's exercise price minus the premium is considered to be the stock's purchase price. The taxable gain or loss on the stock is determined by the difference between the purchase and sale prices of the stock.

Consider the put with an exercise price of $50 written at $3 when the stock price is $52. Suppose the stock price goes to $46 and the put is exercised. The put writer is considered to have purchased the stock for $50 – $3, or $47. If the investor later sells the stock for $55, the taxable gain is $8 and the tax is $8(0.28) = $2.24.

Taxation of Non-Equity Options

Index options, debt options, and foreign currency options have a special tax status. At the end of the calendar year, all realized *and unrealized* gains are taxable. All losses are deductible as previously described. The profits are taxed at a blended rate in which 60 percent are taxed at the long-term capital gains rate and 40 percent are taxed at the short-term capital gains rate, which is the ordinary income rate. For an investor in the 28 percent bracket, this is an effective rate of $0.6(0.20) + 0.4(0.28) = 0.232$.

For example, assume that during the year an investor in the 28 percent bracket had \$1,250 of net profits (profits minus losses) on index options. At the end of the year, the investor holds 1,000 index options worth \$2.25 that previously had been purchased for \$1.75. The unrealized profit is thus \$500. The total taxable profits are \$1,250 + \$500 = \$1,750. The tax is \$1,750(0.6)(0.20) + \$1,750(0.4)(0.28) = \$406.

Wash and Constructive Sales

Option traders should be aware of an important tax condition called the wash sale rule. A wash sale is a transaction in which an investor sells a security at a loss and replaces it with essentially the same security shortly thereafter. Tax laws disallow the deduction of the loss on the sale of the original security. The purpose of the wash sale rule is to prevent investors from taking losses at the end of a calendar year and then immediately replacing the securities. The time period within which the purchase of the security cannot occur is the 61-day period from 30 days before the sale of the stock through 30 days after.

The wash sale rule usually treats a call option as being the same security as the stock. Thus, if the investor sells the stock at a loss and buys a call within the applicable 61-day period, the loss on the stock is not deductible.

In addition, the sale of a call option on a stock owned can, under some circumstances, be treated as the investor effectively having sold the stock, thereby terminating the stock holding period. This is called a constructive sale.

Questions and Problems

1. Which of the following would be a wash sale? Explain.
   a. You buy a stock at \$30. Three weeks later, you sell the stock at \$26. Two weeks later, you buy a call on the stock.
   b. You buy a stock at \$40. One month later, you buy a call on the stock. One week later, you sell the stock for \$38.
   c. You buy a stock for \$40. Three months later, you sell the stock for \$42 and buy a call on the stock.

2. Suppose a stock is priced at \$30 and an eight-month call on the stock with an exercise price of \$25 is priced at \$6. Compute the taxable gain and tax due for each of the following cases, assuming that your tax bracket is 28 percent. Assume 100 shares and 100 calls.
   a. You buy the call. Four months later, the stock is at \$28 and the call is at \$4.50. You then sell the call.
   b. You buy the call. Three months later, the stock is at \$31 and the call is at \$6.50. You then sell the call.
   c. You buy the call. At expiration, the stock is at \$32. You exercise the call and sell the stock a month later for \$35.
   d. You buy the stock and write the call. You hold the position until expiration, whereupon the stock is at \$28.
   e. You write the call. Two months later, the stock is at \$28 and the call is at \$3.50. You buy back the call.

3. Consider an index option. The index is at 425.48, and a two-month call with an exercise price of 425 is priced at $15. You are in the 31 percent tax bracket. Compute the after-tax profit for the following cases. Assume 100 calls.

a. You buy the call. One month later, the index is at 428 and the call is at $12. You sell the call.

b. You buy the call and hold it until expiration, whereupon the index is at 441.35. You exercise the call.

c. You hold the call until expiration, when the index is at 417.15.

d. How will your answers in a and b be affected if the option positions are not closed out by the end of the year?

3

CHAPTER

PRINCIPLES OF OPTION PRICING

This chapter identifies certain factors and shows why they affect an option's price. It examines option boundary conditions—rules that characterize rational option prices. Then it explores the relationship between options that differ by exercise price alone and those that differ only by time to expiration. Finally, the chapter discusses how put and call prices are related as well as several other important principles. An overriding point throughout this chapter, and indeed throughout this entire book, is that arbitrage opportunities are quickly eliminated by investors.

Suppose that an individual offers you the following proposition. You can play a game called Game I in which you draw a ball from a hat known to contain three red balls and three blue balls. If you draw a red ball, you receive nothing; if you draw a blue ball, you receive \$10. Will you play? Because the individual did not mention an entry fee, most people will play. You incur no cash outlay up front and have the opportunity to earn \$10. Of course, this opportunity is too good to be true and only an irrational person would make such an offer without charging an entry fee.

Now suppose that a fair fee to play Game I is \$4. Consider a new game called Game II. The person offers to pay you \$20 if you draw a blue ball and nothing if you draw a red ball. Will the entry fee be higher or lower? If you draw a red ball, you receive the same payoff as in Game I; if you draw a blue ball, you receive a higher payoff than in Game I. You should be willing to pay more to play Game II because these payoffs dominate those of Game I.

From these simple games and opportunities, it is easy to see some of the basic principles of how rational people behave when faced with risky situations. The collective behavior of rational investors operates in an identical manner to determine the fundamental principles of option pricing. As you read the various examples in this chapter in which arbitrage is used to establish fundamental rules about option pricing, keep in mind the similarity of the investment situation to the games just described. In so doing, the rational result should become clear to you.

In this chapter, we do not derive the exact price of an option; rather, we confine the discussion to identifying upper and lower limits and factors that influence an option's price. Chapters 4 and 5 explain how the exact option price is determined.

BASIC NOTATION AND TERMINOLOGY

The following symbols are used throughout the book:

S_0 = stock price today (time 0 = today)
X = exercise price
T = time to expiration as defined below
r = risk-free rate as defined below
S_T = stock price at option's expiration; that is, after the passage of a period of time of length T
$C(S_0,T,X)$ = price of a call option in which the stock price is S_0, the time to expiration is T, and the exercise price is X

$P(S_0,T,X)$ = price of a put option in which the stock price is S_0, the time to expiration is T, and the exercise price is X

In some situations, we may need to distinguish an American call from a European call. If so, the call price will be denoted as either $C_a(S_0,T,X)$ or $C_e(S_0,T,X)$ for the American and European calls, respectively. If there is no a or e subscript, the call can be either an American or a European call. In the case where two options differ only by exercise price, the notations $C(S_0,T,X_1)$ and $C(S_0,T,X_2)$ will identify the prices of the calls with X_1 less than X_2. A good way to remember this is to keep in mind that the subscript of the lower exercise price is smaller than that of the higher exercise price. In the case where two options differ only by time to expiration, the times to expiration will be T_1 and T_2, where $T_1 < T_2$. The options' prices will be $C(S_0,T_1,X)$ and $C(S_0,T_2,X)$. Identical adjustments will be made for put option prices.

For most of the examples, we shall assume that the stock pays no dividends. If, during the life of the option, the stock pays dividends of D_1, D_2, . . . , and so forth, then we can make a simple adjustment and obtain similar results. To do so, we simply subtract the present value of the dividends, $\sum_{j=1}^{N} D_j(1+r)^{-tj}$ where there are N dividends and t_j is the time to each ex-dividend day, from the stock price. We assume that the dividends are known ahead of time.

The time to expiration is expressed as a decimal fraction of a year. For example, if the current date is April 9 and the option's expiration date is July 18, we simply count the number of days between these two dates. That would be 21 days remaining in April, 31 in May, 30 in June, and 18 in July for a total of 100 days. The time to expiration therefore would be 100/365 = 0.274.

The risk-free rate, r, is the rate earned on a riskless investment. An example of such an investment is a U.S. Treasury bill, or T-bill. A Treasury bill is a security issued by the U.S. government for purchase by investors. T-bills with original maturities of 91 and 182 days are auctioned by the Federal Reserve each week; T-bills with maturities of 365 days are auctioned every four weeks. All T-bills mature on a Thursday.[1] Because most exchange-traded options expire on Fridays, there is always a T-bill maturing the day before expiration. For other options with maturities of six months or less, there is a T-bill maturing within one week of the option's expiration. The rate of return on a T-bill of comparable maturity would be a proxy for the risk-free rate.[2]

T-bills pay interest not through coupons but by selling at a discount. The T-bill is purchased at less than face value. The difference between the purchase price and the face value is called the discount. If the investor holds the bill to maturity, it is redeemed at face value. Therefore, the discount is the profit earned by the bill holder.

Bid and ask discounts for several T-bills for the business day of May 14 of a particular year are as follows:

Maturity	Bid	Ask
5/20	4.45	4.37
6/17	4.41	4.37
7/15	4.47	4.43

The bid and ask figures are the discounts quoted by dealers trading in Treasury bills. The bid is the discount if one is selling to the dealer, and the ask is the discount if one is buying from the dealer. Bid and ask quotes are reported daily in *The Wall Street Journal.*

[1]If Thursday is a holiday, such as Thanksgiving, the Treasury bill matures on Wednesday of that week.

[2]There is an ongoing debate regarding the appropriate discount rate to use with financial derivatives. Another possible rate to use is the London Interbank Offer Rate (LIBOR) because it is a good proxy for the marginal dealers' cost of funds. We will use the T-bill rate here because this is an introductory book and T-bills are familiar instruments to most students of finance. We will cover the use of LIBOR in later chapters.

In the United States, most exchange-listed stock options expire on the third Friday of the month. In the preceding example, the third Friday of May was May 21. To find an estimate of the T-bill rate, we use the average of the bid and ask discounts, which is $(4.45 + 4.37)/2 = 4.41$. Then we find the discount from par value as $4.41(7/360) = 0.08575$, using the fact that the option has seven days until maturity. Thus, the price is

$$100 - 0.08575 = 99.91425.$$

Note that the price is determined by assuming a 360-day year. This is a long-standing tradition in the financial community, originating from the days before calculators, when bank loans often were for 60, 90, or 180 days. A banker could more easily calculate the discount using the fraction 60/360, 90/360, or 180/360. This tradition survives today.

The yield on our T-bill is based on the assumption of buying it at 99.91425 and hold-ing it for seven days, at which time it will be worth 100.[3] This is a return of (100 = 99.91425)/99.91425 = 0.000858. If we repeated this transaction every seven days for a full year, the return would be

$$(1.000858)^{365/7} - 1 = 0.0457,$$

where 1.000858 is simply 100/99.91425, or one plus the seven-day return. Note that when we annualize the return, we use the full 365-day year. Thus, we would use 4.57 percent as our proxy for the risk-free rate for options expiring on May 21.

To illustrate the principles of option pricing, we shall be using prices for options on DCRB, a fictional large high-tech company traded on NASDAQ. These prices were assumed to be observed on May 14 and are presented in Table 3.1. The May options expire on May 21, the June options expire on June 18, and the July options expire on July 16.

In the U.S. markets, stock prices are quoted in units of \$0.01 and option prices in units of \$0.05 if the option price is below \$3.00. For option prices above \$3.00, the option prices are quoted in \$0.10. The data illustrated in our examples, assume mid-market quotes implying option prices will be quoted in units of \$0.01.

Following the same procedure described for the May T-bill gives us risk-free rates of 4.56 and 4.63 for the June and July expirations, respectively. The times to expiration are 0.0192 (7 days/365) for the May options, 0.0959 (35 days/365) for the June options, and 0.1726 (63 days/365) for the July options.

Table 3.1 DCRB Option Data, May 14

	Calls			Puts		
Exercise Price	May	June	July	May	June	July
120	8.75	15.40	20.90	2.75	9.25	13.65
125	5.75	13.50	18.60	4.60	11.50	16.60
130	3.60	11.35	16.40	7.35	14.25	19.65

Current stock price: 125.94
Expirations: May 21, June 18, July 16

[3]Actually the number of days is counted from the settlement day, which is the second business day after the purchase day. We shall disregard this point in the calculations in this book.

PRINCIPLES OF CALL OPTION PRICING

In this section we formulate rules that enable us to better understand how call options are priced. It is important to keep in mind that our objective is to determine the price of a call option *prior to* its expiration day.

Minimum Value of a Call

A call option is an instrument with limited liability. If the call holder sees that it is advantageous to exercise it, the call will be exercised. If exercising it will decrease the call holder's wealth, the holder will not exercise it. The option cannot have negative value, because the holder cannot be forced to exercise it. Therefore,

$$C(S_0,T,X) \geq 0.$$

For an American call, the statement that a call option has a minimum value of zero is dominated by a much stronger statement:

$$C_a(S_0,T,X) \geq Max(0,S_0 - X).$$

The expression $Max(0,S_0 - X)$ means "Take the maximum value of the two arguments, zero or $S_0 - X$."

The minimum value of an option is called its intrinsic value, sometimes referred to as parity value, parity, or exercise value. Intrinsic value, which is positive for in-the-money calls and zero for out-of-the-money calls, is the value the call holder receives from exercising the option and the value the call writer gives up when the option is exercised. Note that we are not concerned about the appropriateness of immediately exercising the option; we note only that one could do so if a profit opportunity were available.

Table 3.2 Intrinsic Values and Time Values of DCRB Calls

Exercise Price	Intrinsic Value	Time Value		
		May	June	July
120	5.94	2.81	9.46	14.96
125	0.94	4.81	12.56	17.66
130	0.00	3.60	11.35	16.40

To prove the intrinsic value rule, consider the DCRB June 120 call. The stock price is $125.94, and the exercise price is $120. Evaluating the expression gives Max(0, 125.94 − 120) = 5.94. Now, consider what would happen if the call were priced at less than $5.94—say, $3. An option trader could buy the call for $3, exercise it—which would entail purchasing the stock for $120—and then sell the stock for $125.94. This arbitrage transaction would net an immediate riskless profit of $2.94 on each share.[4] All investors would do this, which would drive up the option's price. When the price of the option reached $5.94, the transaction would no longer be profitable. Thus, $5.94 is the minimum price of the call.

What if the exercise price exceeds the stock price, as do the options with an exercise price of $130? Then Max(0, 125.94 − 130) = 0, and the minimum value will be zero.

Now look at all of the DCRB calls. Those with an exercise price of $120 have a minimum value of Max(0, 125.94 − 120) = 5.94. All three calls with an exercise price of $120 indeed have prices of no less than $5.94. The calls with an exercise price of $125 have minimum values of Max(0, 125.94 − 125) = 0.94 and are priced at no less than 0.94. The calls with an exercise price of 130 have minimum values of Max (0, 125.94 − 130) = 0. All those options obviously have nonnegative values. Thus, all the DCRB call options conform to the intrinsic value rule. In fact, extensive empirical testing has revealed that options in general conform quite closely to the rule.

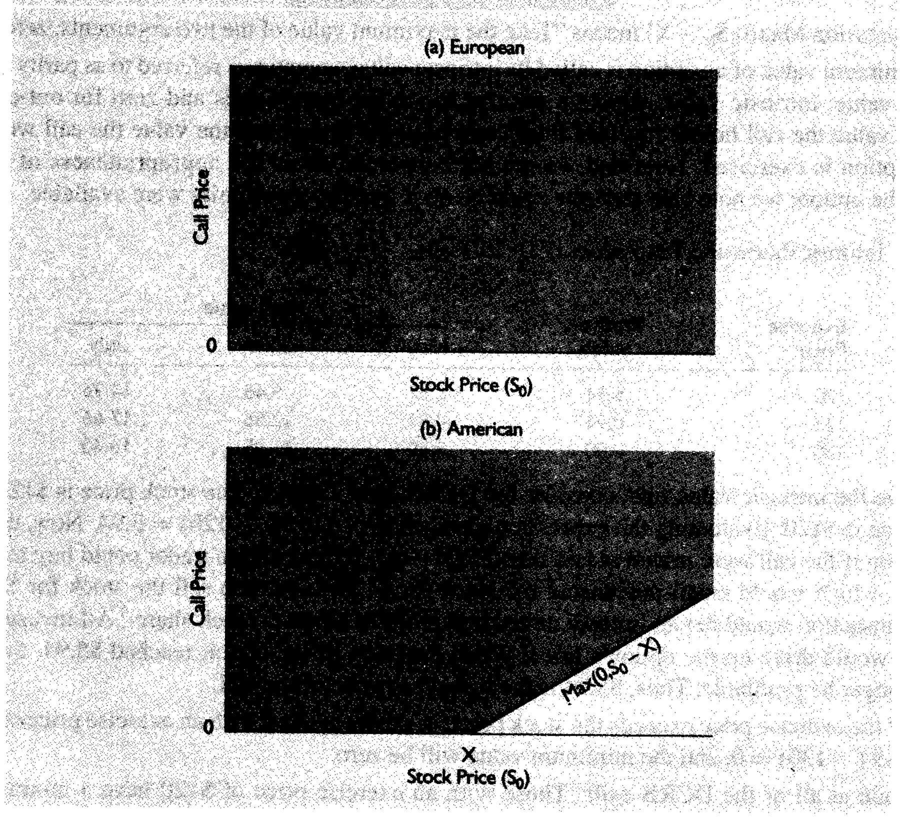
[4]Actually, it would not be necessary to sell the stock. In the absence of transaction costs, it is immaterial whether one holds the stock—an asset valued at $125.94—or converts it to another asset—cash—worth $125.94. The wealth is the same, $125.94, in either case.

The intrinsic value concept applies only to an American call, because a European call can be exercised only on the expiration day. If the price of a European call were less than $Max(0, S_0 - X)$, the inability to exercise it would prevent traders from engaging in the aforementioned arbitrage that would drive up the call's price.

The price of an American call normally exceeds its intrinsic value. The difference between the price and the intrinsic value is called the time value or speculative value of the call, which is defined as $C_a(S_0,T,X) - Max(0, S_0 - X)$. The time value reflects what traders are willing to pay for the uncertainty of the underlying stock. Table 3.2 presents the intrinsic values and time values of the DCRB calls. Note that the time values increase with the time to expiration.

Figure 3.1 illustrates what we have established so far. The call price lies in the shaded area. Figure 3.1(a) illustrates the European call price can lie in the entire area, whereas the American call price lies in a smaller area. This does not mean that the American call price is less than the European call price but only that its range of possible values is narrower.

FIGURE 3.1 Minimum Values of European and American Calls



Maximum Value of a Call

A call option also has a maximum value:

$$C(S_0,T,X) \leq S_0.$$

The call is a conduit through which an investor can obtain the stock. The most one can expect to gain from the call is the stock's value less the exercise price. Even if the exercise price were zero, no one would pay more for the call than for the stock. However, one call that is worth the stock price is one with an infinite maturity. It is obvious that all the DCRB calls are worth no more than the value of the stock.

Figure 3.2 adds the maximum value rule to Figure 3.1. Notice that the maximum value rule has significantly reduced the range of possible option values.

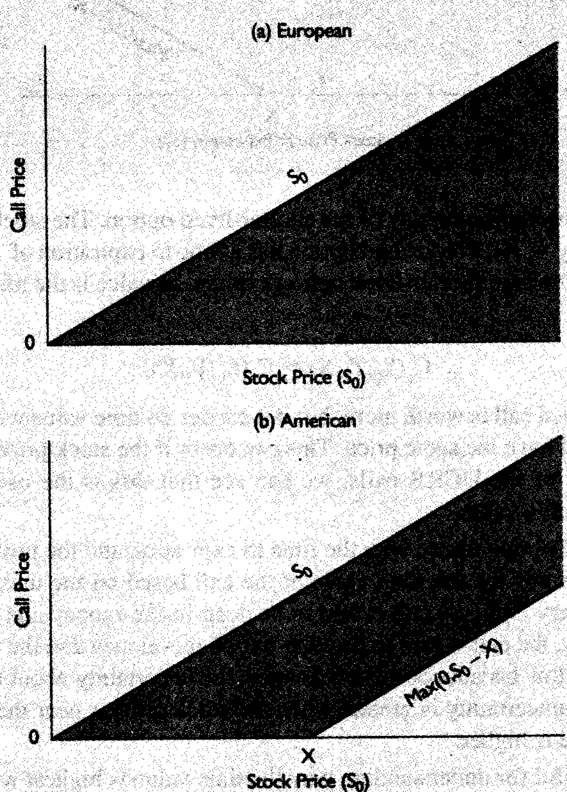
Value of a Call at Expiration

The price of a call at expiration is given as

$$C(S_T,0,X) = Max(0,S_T - X).$$

Because no time remains in the option's life, the call price contains no time value. The prospect of future stock price increases is irrelevant to the price of the expiring option, which will be simply its intrinsic value.[5]

FIGURE 3.2 Minimum and Maximum Values of European and American Calls



At expiration, an American option and a European option are identical instruments. Therefore, this rule holds for both types of options.

Figure 3.3 illustrates the value of the call at expiration. This is one situation in which the value of the call is unambiguous. But do not confuse this with our ultimate objective, which is to find the value of the call *prior to* expiration.

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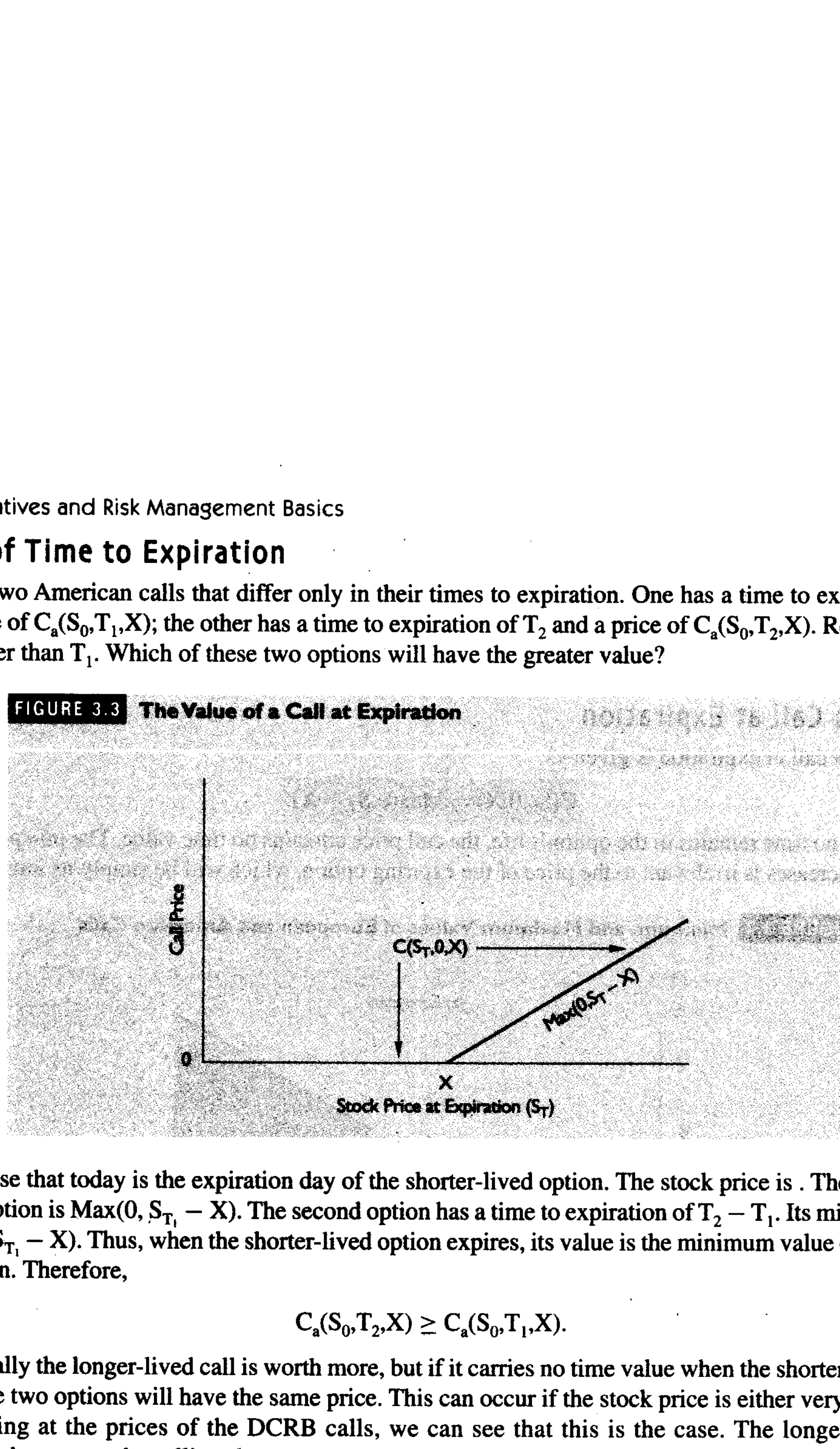
[5]Because of the transaction cost of exercising the option, it could be worth slightly less than the intrinsic value.

Effect of Time to Expiration

Consider two American calls that differ only in their times to expiration. One has a time to expiration of T_1 and a price of $C_a(S_0,T_1,X)$; the other has a time to expiration of T_2 and a price of $C_a(S_0,T_2,X)$. Remember that T_2 is greater than T_1. Which of these two options will have the greater value?

FIGURE 3.3 The Value of a Call at Expiration



Suppose that today is the expiration day of the shorter-lived option. The stock price is . The value of the expiring option is Max(0, $S_{T_1} - X$). The second option has a time to expiration of $T_2 - T_1$. Its minimum value is Max(0, $S_{T_1} - X$). Thus, when the shorter-lived option expires, its value is the minimum value of the longer-lived option. Therefore,

$$C_a(S_0,T_2,X) \geq C_a(S_0,T_1,X).$$

Normally the longer-lived call is worth more, but if it carries no time value when the shorter-lived option expires, the two options will have the same price. This can occur if the stock price is either very high or very low. Looking at the prices of the DCRB calls, we can see that this is the case. The longer the time to expiration, the greater the call's value.

The time value of a call option varies with the time to expiration and the proximity of the stock price to the exercise price. Investors pay for the time value of the call based on the uncertainty of the future stock price. If the stock price is very high, the call is said to be deep-in-the-money and the time value will be low. If the stock price is very low, the call is said to be deep-out-of-the-money and the time value likewise will be low. The time value will be low because at these extremes, the uncertainty about the call expiring in- or out-of-the-money is lower. The uncertainty is greater when the stock price is near the exercise price, and it is at this point that the time value is higher.

A simple analogy is useful for understanding why the time value is highest when the stock price is close to the exercise price. Suppose that you were watching an important basketball game between the New York Knicks and the Boston Celtics. Let us assume that you enjoy a good game but have no sentimental attachment to either team. It is now halftime. Suppose that New York is ahead 65–38. How interesting is the game now? How probable is it that New York will win? Are you likely to keep watching if there is something more interesting to do with your time? But suppose, instead, that the score is tied at 55–55. Isn't the game much more interesting now? Aren't you more likely to watch the second half?

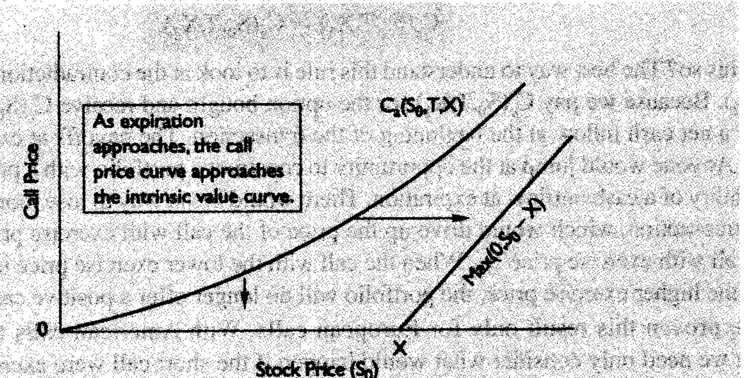
An option with the stock price near the exercise price is like a close game. A deep-in- or out-of-the-money option is like a game with one team well ahead. A close game and an option nearly at-the-money are situations to which people are more willing to allocate scarce resources—for example, time to watch the game or money to buy the option.

These properties of the time value and our previous results enable us to get a general idea of what the call price looks like relative to the stock price. Figure 3.4 illustrates this point for American calls. The curved line is the price of the call, which lies above the intrinsic value, Max(0, S_0 − X). As expiration approaches, the call price loses its time value, a process called time value decay, and the curve moves gradually toward the intrinsic value. At expiration, the call price curve collapses onto the intrinsic value and the curve looks exactly like Figure 3.3.

The time value effect is evident in the DCRB calls. Note that in Table 3.2, for a given exercise price, the time values increase with the time to expiration. For a given time to expiration, the time values are highest for the calls with an exercise price of 125, the exercise price closest to the stock price.

The relationship between time to expiration and option price also holds for European calls. Nevertheless, we cannot yet formally accept this as fact, because we have not yet established a minimum value for a European call. That will come a bit later.

FIGURE 3.4 The Price Curve for American Calls



Effect of Exercise Price

Effect on Option Value Consider two European calls that are identical in all respects except that the exercise price of one is X_1 and that of the other is X_2. Recall that X_2 is greater than X_1. We want to know which price is greater $C_e(S_0,T,X_1)$ or $C_e(S_0,T,X_2)$.

Now consider two portfolios, A and B. Portfolio A consists of a long position in the call with the exercise price of X_1 and a short position in the call with the exercise price of X_2. This type of portfolio is called a money spread and is discussed further in Chapter 7. Because we pay $C_e(S_0,T,X_1)$ and receive $C_e(S_0,T,X_2)$, this portfolio will have an initial value of $C_e(S_0,T,X_1) - C_e(S_0,T,X_2)$. We do not yet know whether the initial value is positive or negative; that will depend on which option price is higher.

Portfolio B consists simply of risk-free bonds with a face value of $X_2 - X_1$. These bonds should be considered as pure discount instruments, like Treasury bills, and as maturing at the options' expiration. Thus, the value of this portfolio is the present value of the bonds' face value, or simply $(X_2 - X_1)(1 + r)^{-T}$.[6]

For the time being, we shall concentrate on portfolio A. First, we need to determine the portfolio's value at expiration. The value of any portfolio will be its cash flow or payoff when the options expire, contingent on the stock price at expiration, S_T. The stock price at expiration has three possible ranges: (1) $S_T < X_1$; (2) $X_1 \leq S_T < X_2$; or (3) $X_1 < X_2 \leq S_T$. Table 3.3 illustrates the values of portfolios A and B at expiration.

[6]Note that the time to expiration can be denoted as T. Technically it is the difference between time T and today, time 0, so we just use T − 0 = T.

Table 3.3 The Effect of Exercise Price on Call Value: Payoffs at Expiration of Portfolios A and B

		Payoffs from Portfolio Given Stock Price at Expiration		
Portfolio	Current Value	$S_T < X_1$	$X_1 \leq S_T < X_2$	$X_1 < X_2 \leq S_T$
A	$+C_e(S_0,T,X_1)$	0	$S_T - X_1$	$S_T - X_1$
	$-C_e(S_0,T,X_2)$	0	0	$-S_T + X_2$
		0	$S_T - X_1 \geq 0$	$X_2 - X_1 > 0$
B	$(X_2 - X_1)(1 + r)^{-T}$	$X_2 - X_1 > 0$	$X_2 - X_1 > 0$	$X_2 - X_1 > 0$

When S_T is greater than X_1, the call option with exercise price X_1 will be worth $S_T - X_1$. If S_T exceeds X_2, the call option with exercise price X_2 will be worth $S_T - X_2$. However, we are short the option with exercise price X_2. Because the buyer receives a payoff of $S_T - X_2$ when this option expires in-the-money, the writer has a payoff of $-S_T + X_2$. Adding the payoffs from the two options shows that portfolio A will always produce a payoff of no less than zero and, in some cases, more than zero. Therefore,

$$C_e(S_0,T,X_1) \geq C_e(S_0,T,X_2).$$

Why is this so? The best way to understand this rule is to look at the contradiction. Suppose that $C_e(S_0,T,X_1) < C_e(S_0,T,X_2)$. Because we pay $C_e(S_0,T,X_1)$ for the option bought and receive $C_e(S_0,T,X_2)$ for the option sold, we will have a net cash inflow at the beginning of the transaction. The payoffs at expiration were shown to be nonnegative. Anyone would jump at the opportunity to construct a portfolio with a positive cash inflow up front and no possibility of a cash outflow at expiration. There would be no way to lose money. Everyone would try to execute this transaction, which would drive up the price of the call with exercise price X_1 and drive down the price of the call with exercise price X_2. When the call with the lower exercise price is worth at least as much as the call with the higher exercise price, the portfolio will no longer offer a positive cash flow up front.

We have proven this result only for European calls. With American calls the long call need not be exercised, so we need only consider what would happen if the short call were exercised early.

Suppose that the stock price prior to expiration is S_t and exceeds X_2. For whatever reason, the short call is exercised. This produces a negative cash flow, $-(S_t - X_2)$. The trader then exercises the long call, which produces a positive cash flow of $S_t - X_1$. The sum of these two cash flows is $X_2 - X_1$, which is positive because $X_2 > X_1$.

Thus, early exercise will not generate a negative cash flow. Portfolio A therefore will never produce a negative cash flow at expiration, even if the options are American calls. Thus, our result holds for American calls as well as for European calls.

Note that this result shows that the price of the call with the lower exercise price cannot be less than that of the call with the higher exercise price. However, the two call prices conceivably could be equal. That could occur if the stock price were very low, in which case both calls would be deep-out-of-the-money. Neither would be expected to expire in-the-money, and both would have an intrinsic value of zero. Therefore, both prices could be approximately zero. However, with a very high stock price, the call with the lower exercise price would carry a greater intrinsic value, and thus its price would be higher than that of the call with the higher exercise price.

The prices of the DCRB calls adhere to the predicted relationships. The lower the exercise price, the higher the call option price.

Limits on the Difference in Premiums Now compare the results of portfolio A with those of portfolio B. Note that in Table 3.3, portfolio B's return is never less than portfolio A's. Therefore, investors would not pay less for portfolio B than for portfolio A. The price of portfolio A is $C_e(S_0,T,X_1) - C_e(S_0,T,X_2)$, the price of the option purchased minus the price of the option sold. The price of portfolio B is $(X_2 - X_1)(1 + r)^{-T}$, the present value of the bond's face value. Thus,

$$(X_2 - X_1)(1 + r)^{-T} \geq C_e(S_0, T, X_1) - C_e(S_0, T, X_2).$$

A related and useful statement is

$$X_2 - X_1 \geq C_e(S_0, T, X_1) - C_e(S_0, T, X_2).$$

This follows because the difference in the exercise prices is greater than the present value of the difference in the exercise prices. For two options differing only in exercise price, the difference in the premiums cannot exceed the difference in the exercise prices.

The intuition behind this result is simple: The advantage of buying an option with a lower exercise price over one with a higher exercise price will not be more than the difference in the exercise prices. For example, if you own the DCRB June 125 call and are considering replacing it with the June 120 call, the most you can gain by the switch is \$5. Therefore, you would not pay more than an additional \$5 for the 120 over the 125. This result will be useful in Chapter 7, where we discuss spread strategies.

For American calls, the call with the lower exercise price is worth at least as much as the call with the higher exercise price. However, the statement that the difference in premiums cannot exceed the present value of the difference in exercise prices does not hold for the American call. If both calls are exercised at time t before expiration and the payoff of $X_2 - X_1$ is invested in risk-free bonds, portfolio A's return will be $(X_2 - X_1)(1 + r)^{(T-t)}$, which will exceed portfolio B's return of $X_2 - X_1$. Thus, portfolio B will not always outperform or match portfolio A.

If, however, the bonds purchased for portfolio B have a face value of $(X_2 - X_1)(1 + r)^T$, and thus a present value of $X_2 - X_1$, portfolio B will always outperform portfolio A. In that case, the current value of portfolio A cannot exceed that of portfolio B. Accordingly, we can state that for American calls

$$X_2 - X_1 \geq C_a(S_0,T,X_1) - C_a(S_0,T,X_2).$$

Table 3.4 presents the appropriate calculations for examining these properties on the DCRB calls. Consider the May 120 and 125 calls. The difference in their premiums is 3.

Table 3.4 The Relationship between Exercise Price and Call Price for DCRB Calls

Exercise Prices	Exercise Price Difference	Difference between Call Prices (Present Value of Difference between Exercise Prices in Parentheses) May	June	July
120, 125	5	3.00 (4.9957)	1.90 (4.9787)	2.30 (4.9611)
120, 130	10	5.15 (9.9914)	4.05 (9.9573)	4.50 (9.9222)
125, 130	5	2.15 (4.9957)	2.15 (4.9787)	2.20 (4.9611)

Note: Risk-free rates are 4.57% (May), 4.56% (June), and 4.63% (July); times to expiration are 0.0192 (May), 0.0959 (June), and 0.1726 (July).

The present value of the difference in exercise prices is $5(1.0457)^{-0.0192} = 4.9957$. The remaining combinations are calculated similarly using the appropriate risk-free rates and times to expiration for those options. Because these are American calls, the difference in their prices must be no greater than the difference in their exercise prices. As Table 3.4 shows, all the calls conform to this condition. In addition, all the differences in the call prices are less than the present value of the difference between the exercise prices. Remember that this result need not hold for American calls because they can be exercised early.

Lower Bound of a European Call

We know that for an American call,

$$C_a(S_0,T,X) \geq Max(0,S_0 - X).$$

Because of the requirement that immediate exercise be possible, we were unable to make such a statement for a European call. We can, however, develop a lower bound for a European call that will be higher than the intrinsic value of an American call.

Again consider two portfolios, A and B. Portfolio A consists of a single share of stock currently priced at S_0, while portfolio B contains a European call priced at $C_e(S_0,T,X)$ and risk-free bonds with a face value of X and, therefore, a present value of $X(1 + r)^T$. The current value of this portfolio is thus $C_e(S_0,T,X) + X(1 + r)^{-T}$. The payoffs from these two portfolios are shown in Table 3.5.

As the table shows, the return on portfolio B is always at least as large as that of portfolio A and sometimes larger. Investors will recognize this fact and price portfolio B at a value at least as great as portfolio A's; that is,

$$C_e(S_0,T,X) + X(1 + r)^{-T} \geq S_0.$$

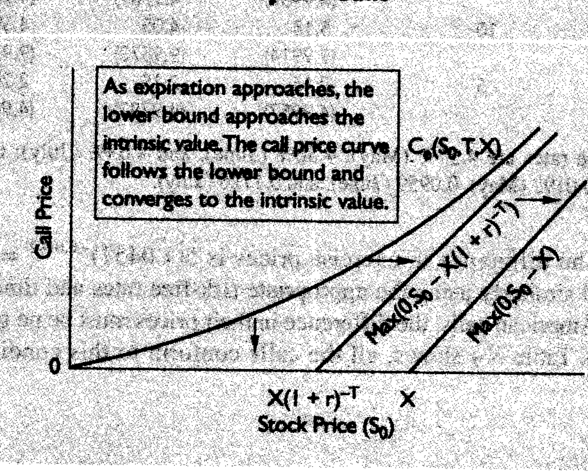
Rearranging this expression gives

$$C_e(S_0,T,X) \geq S_0 - X(1 + r)^{-T}.$$

Table 3.5 The Lower Bound of a European Call: Payoffs at Expiration of Portfolios A and B

Portfolio	Current Value	Payoffs from Portfolio Given Stock Price at Expiration	
		$S_T \leq X$	$S_T > X$
A	S_0	S_T	S_T
B	$C_e(S_0,T,X) + X(1 + r)^{-T}$	X	$(S_T - X) + X = S_T$

FIGURE 3.5 The Price Curve for European Calls



If $S_0 - X(1 + r)^{-T}$ is negative, we invoke the rule that the minimum value of a call is zero. Combining these results gives us a lower bound of

$$C_e(S_0,T,X) \geq \text{Max}[0, S_0 - X(1 + r)^{-T}].$$

If the call price is less than the stock price minus the present value of the exercise price, we can construct an arbitrage portfolio. We buy the call and risk-free bonds and sell short the stock. This portfolio has a positive initial cash flow, because the call price plus the bond price is less than the stock price. At expiration, the payoff is $X - S_T$ if $X > S_T$ and zero otherwise. The portfolio has a positive cash flow today and either a zero or positive cash flow at expiration. Again there is no way to lose money.

Figure 3.5 shows this result for European calls. The curved line is the call price, which must lie above the lower bound. As expiration approaches, the time to expiration decreases such that the lower bound moves to the right. The time value decreases on the option as well and follows the lower bound, with all eventually converging to the intrinsic value, $\text{Max}(0, S_0 - X)$, at expiration.

When we showed that the intrinsic value of an American call is $\text{Max}(0, S_0 - X)$, we noted that the inability to exercise early prevents this result from holding for a European call. Now we can see that this limitation is of no consequence. Because the present value of the exercise price is less than the exercise price itself, the lower bound of a European call is greater than the intrinsic value of an American call.

In the earlier description of how time to expiration affects the price of an American call, we could not draw the same relationship for a European call. Now we can. Consider two European calls that differ by their times to expiration, T_1 and T_2. Their prices are $C_e(S_0,T_1,X)$ and $C_e(S_0,T_2,X)$, respectively. At time T_1, the shorter-lived option expires and is worth $\text{Max}(0,S_{T_1} - X)$. The minimum value of the longer-lived option is $\text{Max}(0,S_{T_1} - X(1 + r)^{-(T_2-T_1)})$. Thus, the value of the shorter-lived option is less than the lower bound of the longer-lived option. Therefore, the longer-lived call must be priced at least as high as the shorter-lived call.

We should note that if the stock pays dividends such that the stock price minus the present value of the dividends is $S_0' = S_0 - \sum_{j=1}^{N} D_j(1+r)^{-t_j}$, then the lower bound is restated as

$$C_e(S_0,T,X) \geq \text{Max}[0,S_0' - X(1 + r)^{-T}].$$

Suppose that instead of options on stock, the option is on a currency. The exchange rate is represented by S_0, but we must recognize that the currency pays interest at the discrete compounding rate of ρ. This is the foreign risk-free rate, which is obtained in the same manner we did for Treasury bills. If we buy one unit of the currency, it will grow to $(1 + \rho)^T$ units at time T. Let us redefine portfolio A to consist of $(1 + \rho)^{-T}$ units of the currency. Because of the accumulation of interest, the $(1 + \rho)^{-T}$ units of the currency will grow to $(1 + \rho)^{-T}(1 + \rho)^T = 1$ unit of the currency. Then, the payoffs are the same as those in Table 3.5. The only difference in this case is that the initial value of portfolio A is now defined as $S_0(1 + \rho)^{-T}$. The lower bound of the call then becomes

$$C_e(S_0,T,X) \geq \text{Max}[0,S_0(1 + \rho)^{-T} - X(1 + r)^{-T}].$$

The similarity between the interest paid on a currency and the dividends on a stock should be apparent. In each case, the asset makes payments to the holder. To establish the lower bound, we must remove these payments from the value of the asset.

We should note that some people view the lower bound for a European option as a kind of adjusted intrinsic value. The price of the stock or currency is, by definition, the present value of its value at expiration. The present value of the exercise price is obviously the exercise price discounted from the expiration. With that in mind, some people prefer to define an at-the-money option as the condition that the stock or currency price, adjusted for dividends or interest, equals the *present value* of the exercise price, instead of just the exercise price. Then the time value would be defined as the option price minus the lower bound. We do not use those definitions in this book, but do not be surprised to encounter them in some other books.

American Call Versus European Call

Many of the results presented so far apply only to European calls. For example, we restricted the derivation of the lower bound to European calls. That is because an early exercise of an American call can negate the cash flows expected from the portfolio at expiration.

In many cases, however, American calls behave exactly like European calls. In fact, an American call can be viewed as a European call with the added feature that it can be exercised early. Since exercising an option is never mandatory,

$$C_a(S_0,T,X) \geq C_e(S_0,T,X).$$

We already proved that the minimum value of an American call is $\text{Max}(0, S_0 - X)$ while the lower bound of a European call is $\text{Max}[0, S_0 - X(1 + r)^{-T}]$. Because $S_0 - X(1 + r)^{-T}$ is greater than $S_0 - X$; as shown in Figure 3.5, the lower bound value of the American call must also be $\text{Max}[0, S_0 - X(1 + r)^{-T}]$.

Let us now examine the DCRB options to determine whether their prices exceed the lower boundary. Table 3.6 presents the lower bound of each DCRB call. To see how the computations are performed, take the May 120 call. The time to expiration is 0.0192, and the risk-free rate is 0.0457. Thus,

$$\text{Max}[0, S_0 - X(1 + r)^{-T}] = \text{Max}[0, 125.94 - 120(1.0457)^{-0.0192}] = 6.0428.$$

Table 3.6 Lower Bounds of DCRB Calls

	Expiration		
Exercise Price	May	June	July
120	6.0428	6.4520	6.8738
125	1.0471	1.4733	1.9127
130	0.0000	0.0000	0.0000

Note: Risk-free rates are 4.57% (May), 4.56% (June), and 4.63% (July); times to expiration are 0.0192 (May), 0.0959 (June), and 0.1726 (July).

From Table 3.1, the price of the call is \$8.75. Thus, this option does meet the boundary condition. Tables 3.1 and 3.6 reveal that the remaining calls also conform to the lower boundary. In general, studies show that call prices conform closely to the lower bound rule.

With the lower bound of an American call established, we can now examine whether an American call should ever be exercised early. First, assume that the call is in-the-money; otherwise, we would not even be considering exercising it. If the stock price is S_0, exercising the call produces a cash flow of $S_0 - X$. The call's price, however, must be at least $S_0 - X(1 + r)^{-T}$. Since the cash flow from exercising, $S_0 - X$, can never exceed the call's lower bound, $S_0 - X(1 + r)^{-T}$, it will always be better to sell the call in the market. When the transaction cost of exercising is compared to the transaction cost of selling the call, the argument that a call should not be exercised early is strengthened. Thus, if the stock pays no dividends, Figure 3.5 is also the call price curve for American calls.

Although many people find it difficult to believe that an American option on a non-dividend-paying stock would never be exercised early, it is really quite easy to see by analogy. Have you ever subscribed to a magazine and, shortly afterward, received a renewal notice mentioning the attractive rate they could offer you? You have barely begun receiving the magazine, perhaps have not yet decided if you like it, and already you are granted an option to renew. Do you immediately renew? Hardly any rational person would do so. Not only would you be giving the magazine company the interest on your money, you would also be giving up the right to decide later if you wanted to renew. It is almost always the case that you would still have the right to renew much later at the same rate. Likewise, exercising a call option early not only gives the writer the interest on your money, but also throws away the right to decide later if you want to own the stock.

If that argument is not convincing, simply consider this. The urge to exercise a deep-in-the-money call simply because you think the stock has gone as far up as you think it will should be suppressed. If your views on the stock are correct, you will not be any more pleased to own a stock that is not increasing than you would to be holding the call option. The story changes, however, if the stock pays dividends.

Early Exercise of American Calls on Dividend-Paying Stocks

When a company declares a dividend, it specifies that the dividend is payable to all stockholders as of a certain date, called the holder-of-record date. Two business days before the holder-of-record date is the ex-dividend date. To be the stockholder of record by the holder-of-record date, one must buy the stock by the ex-dividend date. The stock price tends to fall by the amount of the dividend on the ex-dividend date.

When a stock goes ex-dividend, the call price drops along with it. The amount by which the call price falls cannot be determined at this point in our understanding of option pricing. Since the call is a means of obtaining the stock, however, its price could never change by more than the stock price change. Thus, the call price will fall by no more than the dividend. An investor could avoid this loss in value by exercising the option immediately before the stock goes ex-dividend. This is the only time the call should be exercised early.

Another way to see that early exercise could occur is to recall that we stated that the lower bound of a European call on a dividend-paying stock is $\text{Max}[0,S_0' - X(1 + r)^{-T}]$ where S_0' is the stock price minus the present value of the dividends. To keep things simple, assume only one dividend of the amount D, and that the stock will go ex-dividend in the next instant. Then S_0' is approximately equal to $S_0 - D$ (since the present value of D is almost D). Since we would consider exercising only in-the-money calls, assume that S_0 exceeds X. Then it is easy to see that $S_0 - X$ could exceed S_0' minus the present value of X. By exercising the option, the call holder obtains the value $S_0 - X$. Suppose that you were holding a European call whose value was only slightly above the lower bound. Then you might wish it were an American call because an American call could be exercised to capture the value $S_0 - X$. If you were holding a European call and wished it were an American call for at least that instant, then it should be obvious that the right to exercise early would have value. So an American call could be worth more than a European call. Note that this does not mean that exercise will definitely occur at the ex-dividend instant. It means only that exercise *could* occur. The value of the right to exercise early is what distinguishes an American call from a European call. That right is worth something only when there are dividends on the stock. A similar argument applies when the underlying is a currency, which pays interest.

There is one other situation in which you can determine that the right to exercise early has no value. If the present value of all the dividends over the life of the option is less than $X(1 - (1 + r)^{-T})$, then the option would never be exercised early because the dividends are not large enough to offset the loss of interest from paying out the exercise price early.

In Chapters 4 and 5 we shall learn about option pricing models that will give us the exact price of the option. We will then be able to see the early exercise value more explicitly.

Effect of Interest Rates

Interest rates affect a call option's price in several ways. First, interest rates affect the lower bound. The lower the interest rate, the lower the lower bound. In the extreme case of a zero interest rate, the lower bound would be the same as the intrinsic value. Nothing special happens to the option price if the interest rate is zero. It will still have value that will be limited by the lower bound/intrinsic value. But there are other more complex effects as well. Perhaps the easiest way to understand the effect of interest rates on calls is to think of a call as a way to purchase stock by paying an amount of money less than the face value of the stock. By paying the call premium, you save the difference between the call price and the exercise price, the price you are willing to pay for the stock. The higher the interest rate, the more interest you can earn on the money you saved by buying the call. Thus, when interest rates increase, calls are more attractive to buyers so they will have higher prices.[7]

Effect of Stock Volatility

One of the basic principles of investor behavior is that individuals prefer less risk to more. For holders of stocks, higher risk means lower value. But higher risk in a stock translates into greater value for a call option on it. This is because greater volatility increases the gains on the call if the stock price increases, because the stock price can then exceed the exercise price by a larger amount. On the other hand, greater volatility means that if the stock price goes down, it can be much lower than the exercise price. To a call holder, however, this does not matter because the potential loss is limited; it is said to be truncated at the exercise price. For example, consider the DCRB July 125 call. Suppose the stock price is equally likely to be at 110, 120, 130, or 140 at expiration. The call, then, is equally likely to be worth 0, 0, 5, and 15 at expiration. Now suppose the stock's volatility increases so that it has an equal chance of being at 100, 120, 130, or 150. From a stockholder's point of view, the stock is far riskier, which is less desirable. From the option holder's perspective, the equally possible option prices at expiration are 0, 0, 5, and 25, which is more desirable. In fact, the option holder will not care how low the stock can go. If the possibility of lower stock prices is accompanied by the possibility of higher stock prices, the option holder will benefit, and the option will be priced higher when the volatility is higher.

Another way to understand the effect of volatility on the call price is to consider the extreme case of zero volatility.[8] If the stock price is less than the exercise price, the absence of volatility guarantees that the option will expire out-of-the-money. No one would pay anything for this option. If the stock price exceeds the exercise price and has zero volatility, it will expire in-the-money and will be worth $S_T - X$ at expiration, where S_T is the future value of S_0. In this case, the call will then simply be a risk-free asset worth S_0 minus the present value of X. Since volatility does not affect the lower bound, the call price and lower bound remain above the intrinsic value, reflecting the fact that the option will still not be exercised until expiration. On the other hand, high volatility is what makes call options attractive, and investors are willing to pay higher premiums on options with greater volatility.

In Chapter 4, we shall begin to explore in more detail how volatility affects option prices. In Chapter 5 we look at volatility in more detail, and we see that volatility is captured by the standard deviation of the stock return.

PRINCIPLES OF PUT OPTION PRICING

Many of the rules applicable to call options apply in a straightforward manner to put options. There are, however, some significant differences.

Minimum Value of a Put

A put is an option to sell a stock. A put holder is not obligated to exercise it and will not do so if exercising will decrease wealth. Thus, a put can never have a negative value:

$$P(S_0,T,X) \geq 0.$$

[7]The exact relationship between interest rates and call prices is more involved than this, however, because the purchase of a call is actually more than just an option to defer purchase of the stock. Holding the call instead of the stock limits your loss to much less than if you had bought the stock. A call is said to have insurance value. Interest rates affect the insurance value of the call in a negative manner, but the overall effect on the call price is still positive. See D. M. Chance, "Translating the Greek: The Real Meaning of Call Option Derivatives," *Financial Analysts Journal* 50 (July–August 1994): 43–49.

[8]To keep this discussion simple, we also assume a zero risk-free rate.

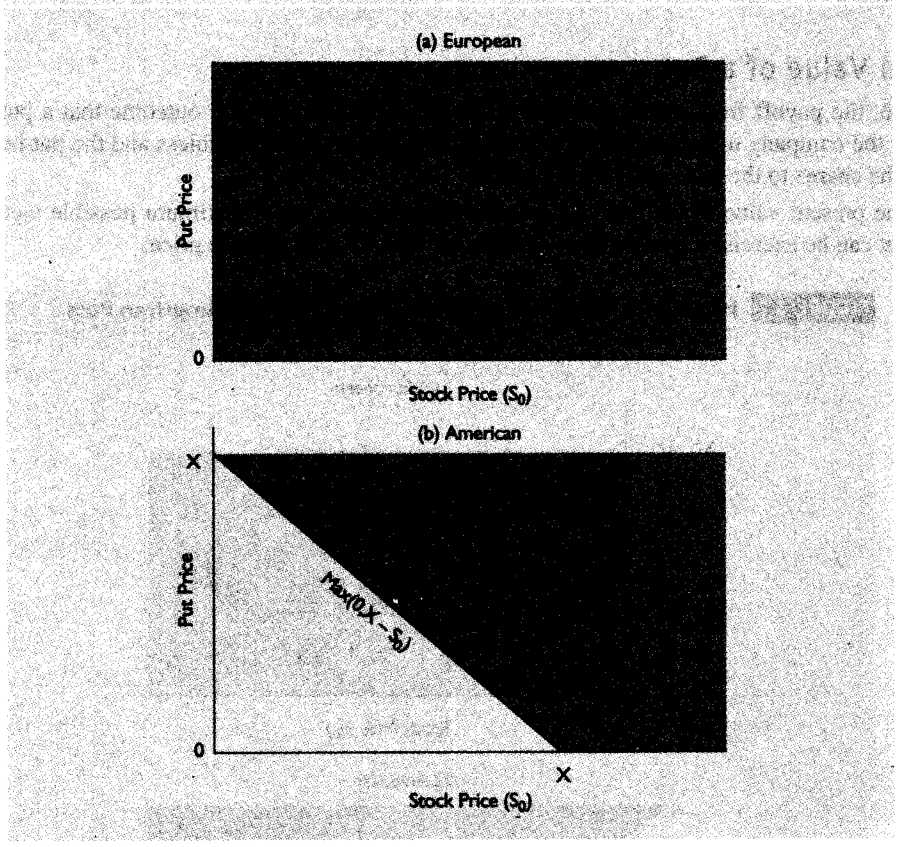
An American put can be exercised early. Therefore,

$$P_a(S_0,T,X) \geq \text{Max}(0,X - S_0).$$

Suppose that the DCRB June 130 put sells for less than $X - S_0$. Let the put sell for \$3. Then it would be worthwhile to buy the stock for \$125.94, buy the put for \$3, and exercise the put. This would net an immediate risk-free profit of \$1.06. The combined actions of all investors conducting this arbitrage would force the put price up to at least \$4.06, the difference between the exercise price and the stock price.

Figure 3.6 illustrates these points for puts. The European put price lies somewhere in the shaded area of graph a. The American put price lies somewhere in the shaded area of graph b.

FIGURE 3.6 Minimum Value of European and American Puts



The value, $\text{Max}(0, X - S_0)$, is called the put's *intrinsic value*. An in-the-money put has a positive intrinsic value, while an out-of-the-money put has an intrinsic value of zero. The difference between the put price and the intrinsic value is the *time value* or *speculative value*. The time value is defined as $P_a(S_0,T,X) - \text{Max}(0, X - S_0)$. As with calls, the time value reflects what an investor is willing to pay for the uncertainty of the final outcome.

Table 3.7 presents the intrinsic values and time values of the DCRB puts. Note how the values increase with the time to expiration.

Table 3.7 Intrinsic Values and Time Values of DCRB Puts

		Time Value		
Exercise Price	Intrinsic Value	May	June	July
120	0.00	2.75	9.25	13.65
125	0.00	4.60	11.50	16.60
130	4.06	3.29	10.19	15.59

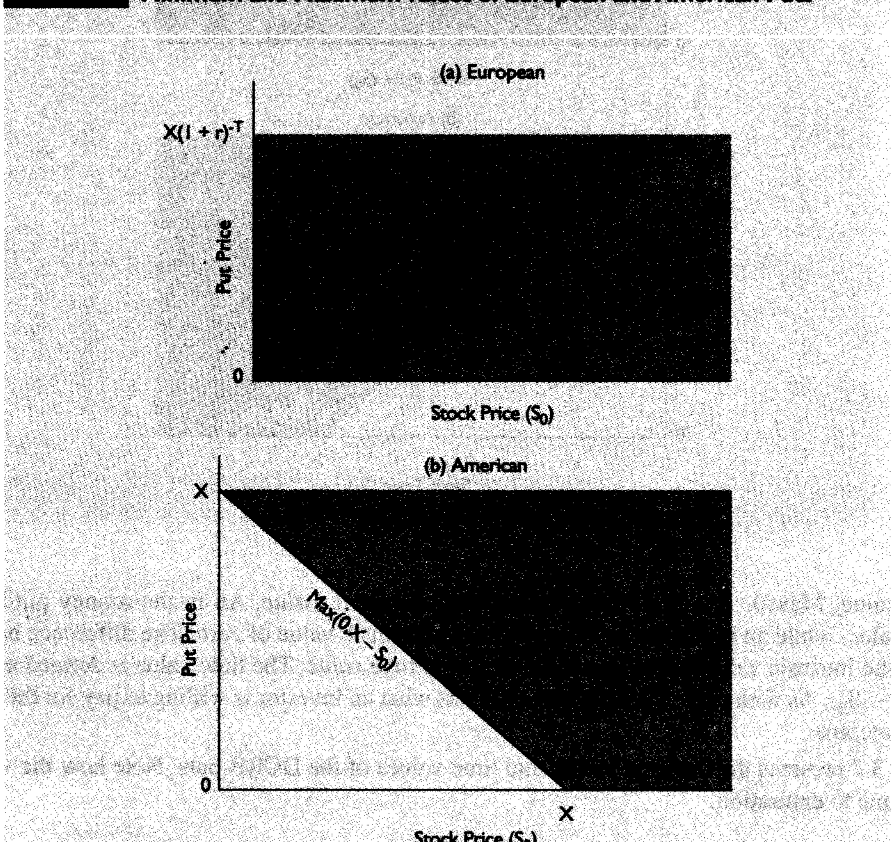
The intrinsic value specification, Max(0, $X - S_0$), does not hold for European puts. That is because the option must be exercisable for an investor to execute the arbitrage transaction previously described. European puts indeed can sell for less than the intrinsic value. Later this will help us understand the early exercise of American puts.

Maximum Value of a Put

At expiration, the payoff from a European put is Max(0, $X - S_T$). The best outcome that a put holder can expect is for the company to go bankrupt. In that case, the stock will be worthless and the put holder will be able to sell the shares to the put writer for X dollars.

Thus, the present value of the exercise price is the European put's maximum possible value. Since an American put can be exercised at any time, its maximum value is the exercise price:

FIGURE 3.7 Minimum and Maximum Values of European and American Puts



$$P_e(S_0,T,X) \leq X(1+r)^{-T},$$
$$P_a(S_0,T,X) \leq X.$$

Figure 3.7 adds the maximum value rule to Figure 3.6. Although the range of possible values is reduced somewhat, there is still a broad range of possible values.

Value of a Put at Expiration

On the put's expiration date, no time value will remain. Expiring American puts therefore are the same as European puts. The value of either type of put must be the intrinsic value. Thus,

$$P(S_T,0,X) = Max(0,X - S_T).$$

If $X > S_T$ and the put price is less than $X - S_T$, investors can buy the put and the stock, and exercise the put for an immediate risk-free profit. If the put expires out-of-the-money ($X < S_T$), it will be worthless. Figure 3.8 illustrates the value of the put at expiration.

Effect of Time to Expiration

Consider two American puts, one with a time to expiration of T_1 and the other with a time to expiration of T_2, where $T_2 > T_1$. Now, assume today is the expiration date of the shorter-lived put. The stock price is . The expiring put is worth $Max(0, X - S_{T_1})$. The other put, which has a remaining time to expiration of $T_2 - T_1$, is worth at least $Max(0, X - S_{T_1})$. Consequently, it must be true that at time 0, we must have,

$$P_a(S_0,T_2,X) \geq P_a(S_0,T_1,X).$$

Note that the two puts could be worth the same; however, this would occur only if both puts were very deep-in- or out-of-the-money. Even then, the longer-lived put would likely be worth a little more than the shorter-lived put. The longer-lived put can do everything the shorter-lived put can do and has an additional period of time in which to increase in value.

The principles that underlie the time value of a put are the same as those that underlie the time value of a call. The time value is largest when the stock price is near the exercise price and smallest when the stock price is either very high or very low relative to the exercise price. With these points in mind, we can now see that the American put price would look like Figure 3.9. As expiration approaches, the put price curve approaches the intrinsic value, which is due to the time value decay. At expiration the put price equals the intrinsic value, as in Figure 3.8.

FIGURE 3.8 The Value of a Put at Expiration

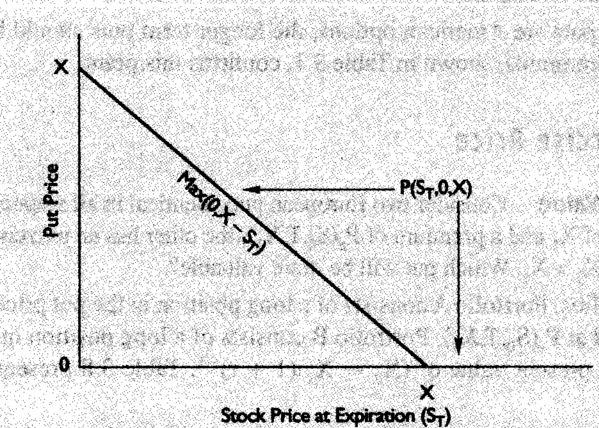


FIGURE 3.9 The Price Curve for American Puts

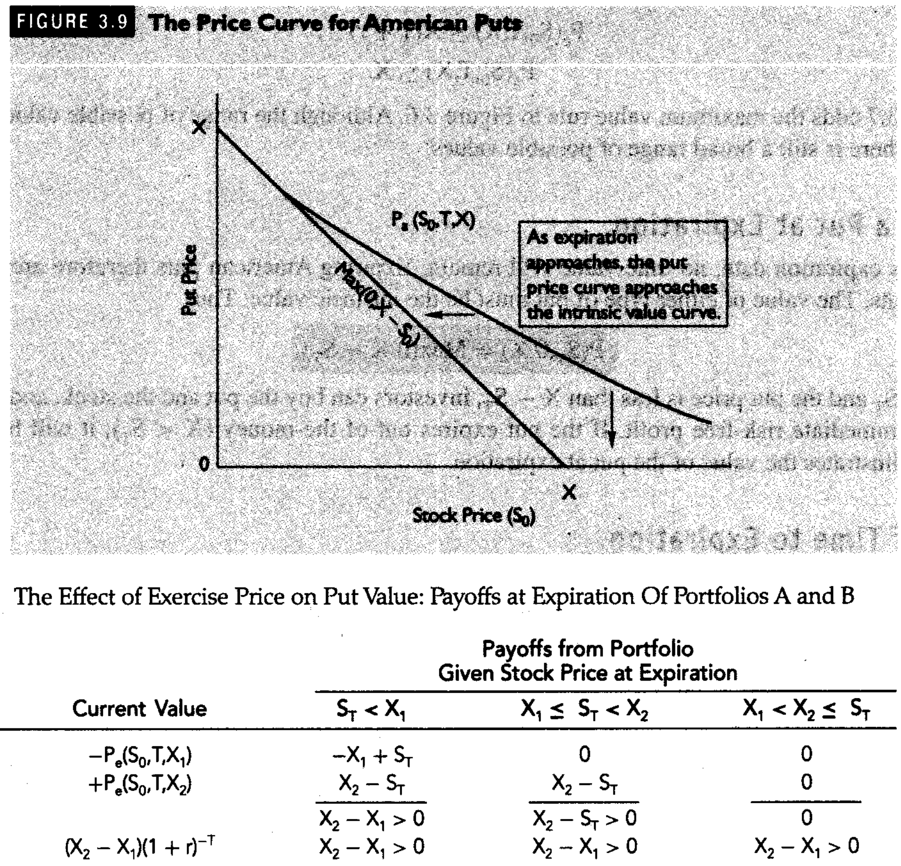


Table 3.8 The Effect of Exercise Price on Put Value: Payoffs at Expiration Of Portfolios A and B

		Payoffs from Portfolio Given Stock Price at Expiration		
Portfolio	Current Value	$S_T < X_1$	$X_1 \leq S_T < X_2$	$X_1 < X_2 \leq S_T$
A	$-P_e(S_0,T,X_1)$	$-X_1 + S_T$	0	0
	$+P_e(S_0,T,X_2)$	$X_2 - S_T$	$X_2 - S_T$	0
		$X_2 - X_1 > 0$	$X_2 - S_T > 0$	0
B	$(X_2 - X_1)(1 + r)^{-T}$	$X_2 - X_1 > 0$	$X_2 - X_1 > 0$	$X_2 - X_1 > 0$

The relationship between time to expiration and put price is more complex for European puts. Think of buying a put as deferring the sale of stock at the exercise price, X. The further into the future the expiration date, the longer the put holder must wait to sell the stock and receive X dollars. This can make the longer-lived European put less valuable than the shorter-lived one. This does not hold for an American put, because the holder can always exercise it and receive X dollars today. For a European put, the longer time to expiration is both an advantage—the greater time value—and a disadvantage—the longer wait to receive the exercise price. The time value effect tends to dominate, however, and in most cases longer-lived puts will be more valuable than shorter-lived puts.

Because the DCRB puts are American options, the longer-term puts should be expected to show higher prices. A check of their premiums, shown in Table 3.1, confirms this point.

The Effect of Exercise Price

The Effect on Option Value Consider two European puts identical in all respects except exercise price. One put has an exercise price of X_1 and a premium of $P_e(S_0,T,X_1)$; the other has an exercise price of X_2 and a premium of $P_e(S_0,T,X_2)$. As before $X_2 > X_1$. Which put will be more valuable?

Consider two portfolios. Portfolio A consists of a long position in the put priced at $P_e(S_0,T,X_2)$ and a short position in the put priced at $P_e(S_0,T,X_1)$. Portfolio B consists of a long position in risk-free bonds with a face value of $X_2 - X_1$ and a present value of $(X_2 - X_1)(1 + r)^{-T}$. Table 3.8 presents these portfolios' payoffs at expiration.

For portfolio A, all outcomes are nonnegative. Because this portfolio cannot produce a cash outflow for the holder, the price of the put purchased must be no less than the price of the put sold; that is,

$$P_e(S_0,T,X_2) \geq P_e(S_0,T,X_1).$$

To understand why this is so, consider what would happen if it were not. Suppose that the price of the put sold were greater than the price of the put purchased. Then an investor would receive more for the put sold than would be paid for the put purchased. That would produce a net positive cash flow up front, and, from Table 3.8, there would be no possibility of having to pay out any cash at expiration. This transaction would be like a loan that need not be paid back. Obviously, this portfolio would be very attractive and would draw the attention of other investors, who would buy the put with the higher exercise price and sell the put with the lower exercise price. This would tend to drive up the price of the former and drive down the price of the latter. The transaction would cease to be attractive when the put with the higher exercise price became at least as valuable as the put with the lower exercise price.

The intuition behind why a put with a higher exercise price is worth more than one with a lower exercise price is quite simple. A put is an option to sell stock at a fixed price. The higher the price at which the put holder can sell the stock, the more attractive the put.

Suppose that these were American puts. In that case, the put with the lower exercise price could be exercised early. For example, let the stock price at time t prior to expiration be S_t, where $S_t < X_1$. Let the option with exercise price X_1 be exercised. Then the investor simply exercises the option with exercise price X_2. The cash flow from these two transactions is $-(X_1 - S_t) + (X_2 - S_t) = X_2 - X_1$, which is positive. Early exercise will not generate a negative cash flow. Thus, our result holds for American puts as well as for European puts.

Limits on the Difference in Premiums Now let us compare the outcomes of portfolios A and B. We see that portfolio B's outcomes are never less than portfolio A's. Therefore, no one would pay more for portfolio A than for portfolio B; that is,

$$(X_2 - X_1)(1 + r)^{-T} \geq P_e(S_0,T,X_2) - P_e(S_0,T,X_1).$$

This result does not, however, hold for American puts. If the puts were American and both were exercised, the investor would receive $X_2 - X_1$ dollars. This amount would be invested in risk-free bonds and would earn interest over the options' remaining lives. At expiration, the investor would have more than $X_2 - X_1$, the payoff from portfolio B.

Since the difference in exercise prices is greater than the present value of the difference in exercise prices, we can state that for European puts,

$$X_2 - X_1 \geq P_e(S_0,T,X_2) - P_e(S_0,T,X_1).$$

This means that the difference in premiums cannot exceed the difference in exercise prices. This result holds for American as well as European puts. To see this, let portfolio B's bonds have a face value of $(X_2 - X_1)(1 + r)^T$ and a present value of $X_2 - X_1$. If early exercise occurred at time t, the most the holder of portfolio A would have at expiration is $(X_2 - X_1)(1 + r)^{T-t}$. The holder of portfolio B would have a larger amount, $(X_2 - X_1)(1 + r)^T$. So again portfolio A would never pay more at expiration than would portfolio B. Therefore, the current value of portfolio A, $P_a(S_0,T,X_2) - P_a(S_0,T,X_1)$, could not exceed the current value of portfolio B, $X_2 - X_1$. Thus,

$$X_2 - X_1 \geq P_a(S_0,T,X_2) - P_a(S_0,T,X_1).$$

Table 3.9 shows the differences between the put premiums and exercise prices for the DCRB puts. Since these are American puts, we would expect only that the difference in their put premiums will not exceed the difference in their exercise prices—which indeed is the case. In addition, the differences in premiums do not exceed the present values of the differences in exercise prices.

Lower Bound of a European Put

We showed that the minimum value of an American put is Max(0, $X - S_0$). This statement does not hold for a European put, because it cannot be exercised early. However, it is possible to derive a positive lower bound for a European put.

Table 3.9 The Relationship between Exercise Price and Put Price for DCRB Puts

		Difference between Put Prices (Present Value of Difference between Exercise Prices in Parentheses)		
Exercise Prices	Exercise Price Difference	May	June	July
120, 125	5	1.85 (4.9957)	2.25 (4.9787)	2.95 (4.9611)
120, 130	10	4.60 (9.9914)	5.00 (9.9573)	6.00 (9.9222)
125, 130	5	2.75 (4.9957)	2.75 (4.9787)	3.05 (4.9611)

Note: Risk-free rates are 4.57% (May), 4.56% (June), and 4.63% (July); times to expiration are 0.0192 (May), 0.0959 (June), and 0.1726 (July).

Again consider two portfolios, A and B. Portfolio A consists of a single share of stock. Portfolio B consists of a short position in a European put priced at $P_e(S_0,T,X)$ and a long position in risk-free bonds with a face value of X and a present value of $X(1 + r)^{-T}$. The payoffs at expiration from these portfolios are shown in Table 3.10.

Table 3.10 Lower Bound of a European Put: Payoffs at Expiration of Portfolios A and B

		Payoffs from Portfolio Given Stock Price at Expiration	
Portfolio	Current Value	$S_T < X$	$S_T \geq X$
A	S_0	S_T	S_T
B	$X(1 + r)^{-T} - P_e(S_0,T,X)$	$X - (X - S_T) = S_T$	X

Portfolio A's outcome is always at least as favorable as portfolio B's. Therefore, no one would be willing to pay more for portfolio B than for portfolio A. Portfolio A's current value must be no less than portfolio B's; that is,

$$S_0 \geq X(1 + r)^{-T} - P_e(S_0,T,X).$$

Rearranging this statement gives

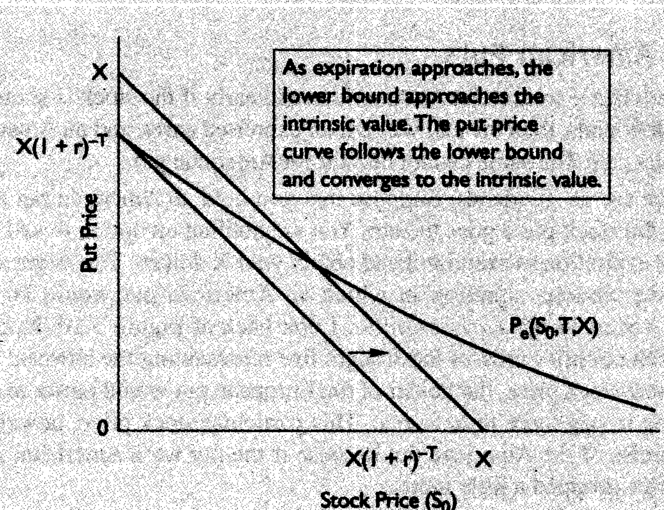
$$P_e(S_0,T,X) \geq X(1 + r)^{-T} - S_0.$$

If the present value of the exercise price is less than the stock price, this lower bound will be negative. Since we know that a put cannot be worth less than zero, we can say that

$$P_e(S_0,T,X) \geq \text{Max}[0, X(1 + r)^{-T} - S_0].$$

Figure 3.10 illustrates these results. The curved line is the European put price, which must lie above the lower bound. As expiration approaches, the time to expiration gets smaller, so the lower bound moves to the right and at expiration converges to the intrinsic value. The put price curve moves down, a result of the loss of its time value as expiration approaches, but is still above the lower bound. At expiration, the put price, lower bound, and intrinsic value are all the same.

FIGURE 3.10 The Price Curve for European Puts



Now let us compare the minimum price of the American put, its intrinsic value of $\text{Max}(0, X - S_0)$, with the lower bound of the European put, $\text{Max}[0, X(1 + r)^{-T} - S_0]$. Since $X - S_0$ is greater than $X(1 + r)^{-T} - S_0$, the American put's intrinsic value is higher than the European put's lower bound. Therefore, the European put's lower bound is irrelevant to the American put price because it is a lower minimum, as seen in Figure 3.10. $\text{Max}[0, X(1 + r)^{-T} - S_0]$ is, however, relevant to the European put's price.

Finally, we can use the lower bound of a European put to examine the effect of time to expiration on the option. Earlier we stated that the direction of this effect is uncertain. Consider two puts with times to expiration of T_1 and T_2, where $T_2 > T_1$. Suppose that we are at time T_1, the stock price is S'_T, and the shorter-lived put is expiring and is worth $\text{Max}(0, X - S'_T)$. The longer-lived put has a remaining life of $T_2 - T_1$ and a lower bound of $\text{Max}[0, X(1 + r)^{-(T_2 - T_1)} - S'_T]$. Although the lower bound of the longer-lived put is less than the shorter-lived put's intrinsic value, the value of the additional time on the former, during which the stock price can move can more than make up the difference. Therefore, we cannot unambiguously tell whether a longer- or shorter-lived European put will be worth more.

If the stock pays dividends such that that is, the stock price minus the present value of the dividends, the rule becomes

$$P_e(S_0,T,X) \geq \text{Max}[0, X(1 + r)^{-T} - S'_0].$$

As we did for a call, if the underlying is a currency, we adjust portfolio A to start off with $(1 + \rho)^{-T}$ units of the currency. This will grow to one unit at T, and the payoffs will be the same as those in Table 3.10. The net effect is that the lower bound for the put is

$$P_e(S_0,T,X) \geq \text{Max}[0, X(1 + r)^{-T} - S_0(1 + \rho)^{-T}].$$

American Put Versus European Put

Every`hing that can be done with a European put can be done with an American put. In addition, an American put can be exercised at any time prior to expiration. Therefore, the American put price must at least equal the European put price; that is,

$$P_a(S_0,T,X) \geq P_e(S_0,T,X).$$

Early Exercise of American Puts

Recall that it might be advisable to exercise an American call early if the stock is about to go ex-dividend. If the stock does not pay dividends, then the call will not be exercised early, and an American call will have the same value as a European call. The same cannot be said for American puts.

Let us suppose there are no dividends. Suppose that you hold an American put and the company goes bankrupt, meaning that the stock price goes to zero. You are holding an option to sell it for X dollars. There is no reason to wait until expiration to exercise it and obtain your X dollars. You might as well exercise it now. Thus, bankruptcy is one obvious situation in which an American put would be exercised early, but, *bankruptcy is not required to justify early exercise.* Look back at Figure 3.10. Note that the curved line representing the European put price crosses the straight line representing the intrinsic value of the American put. That means that at that stock price, the holder of the European put would prefer to have had an American put and would be willing to pay more to have one. This particular stock price, however, is not the one that would trigger early exercise of the American put because if the put were American, it would be optimal to hold it until the stock price dropped a little more.

If the stock pays dividends, it might still be worthwhile to exercise it early, but because dividends drive the stock price down, they make American puts less likely to be exercised early. In fact, if the dividends are sufficiently large, it can sometimes be shown that the put would never be exercised early, thus making it effectively a European put. With currency options, the foreign interest has a similar effect.

At this point in our material, the exact situation at which an American put would be exercised early cannot be specified. We shall cover this topic again in more detail, however, in Chapter 4.

Put-Call Parity

The prices of European puts and calls on the same stock with identical exercise prices and expiration dates have a special relationship. The put price, call price, stock price, exercise price, time to expiration, and risk-free rate are all related by a formula called put-call parity. Let us see how this formula is derived.

Imagine a portfolio, called portfolio A, consisting of one share of stock and a European put. This portfolio will require an investment of $S_0 + P_e(S_0,T,X)$. Now consider a second portfolio, called portfolio B, consisting of a European call with the same exercise price and risk-free pure discount bonds with a face value of X. That portfolio will require an investment of $C_e(S_0,T,X) + X(1 + r)^{-T}$. Now let us look at what happens at the expiration. Table 3.11 presents the outcomes.

The stock is worth S_T regardless of whether S_T is more or less than X. Likewise the risk-free bonds are worth X regardless of the outcome. If S_T exceeds X, the call expires in-the-money and is worth $S_T - X$ and the put expires worthless. If S_T is less than or equal to X, the put expires in-the-money worth

Table 3.11 Put-Call Parity

		Payoffs from Portfolio Given Stock Price at Expiration	
Payoff From	Current Value	$S_T \leq X$	$S_T > X$
A Stock	S_0	S_T	S_T
Put	$P_e(S_0,T,X)$	$X - S_T$	0
		X	S_T
B Call	$C_e(S_T,T,X)$	0	$S_T - X$
Bonds	$X(1 + r)^{-T}$	X	X
		X	S_T

$X - S_T$ and the call expires worthless. The total values of portfolios A and B are equal. Recalling our law of one price from Chapter 1, the current values of the two portfolios must be equal. Thus, we require that

$$S_0 + P_e(S_0,T,X) = C_e(S_0,T,X) + X(1 + r)^{-T}.$$

This statement is referred to as put-call parity and it is probably one of the most important results in understanding options. It says that a share of stock plus a put is equivalent to a call plus risk-free bonds. It shows the relationship between the call and put prices, the stock price, the time to expiration, the risk-free rate, and the exercise price.

Suppose the combination of the put and the stock is worth less than the combination of the call and the bonds. Then you could buy the put and the stock and sell short the call and the bonds. Selling short the call just means to write the call and selling short the bonds simply means to borrow the present value of X and promise to pay back X at the options' expiration. The cash inflow of the value of the call and the bonds would exceed the cash outflow for the put and the stock. At expiration, there would be no cash inflow or outflow because you would have stock worth S_T and owe the principal on the bonds of X, but you would also have a put worth $X - S_T$ and a call worth zero or a call worth $-(S_T - X)$ and a put worth zero. All of this adds up to zero (check it). In other words, you would get some money up front but not have to pay any out at expiration. Since everyone would start doing this transaction, the prices would be forced back in line with the put-call parity equation.

By observing the signs in front of each term, we can easily determine which combinations replicate others. If the sign is positive, we should buy the option, stock, or bond. If it is negative, we should sell. For example, suppose we isolate the call price,

$$C_e(S_0,T,X) = P_e(S_0,T,X) + S_0 - X(1 + r)^{-T}.$$

Then owning a call is equivalent to owning a put, owning the stock, and selling short the bonds (borrowing). If we isolate the put price,

$$P_e(S_0,T,X) = C_e(S_0,T,X) - S_0 + X(1 + r)^{-T}.$$

This means that owning a put is equivalent to owning a call, selling short the stock, and buying the bonds. Likewise, we could isolate the stock or the bonds.

To convince yourself that these combinations on the right-hand side of the equation are equivalent to the combinations on the left-hand side, it would be helpful to set up a table like Table 3.11. Analyze the outcomes at expiration, and you will see that the various combinations on the right-hand side do indeed replicate the positions taken on the left-hand side.

If the stock pays dividends, once again, we simply insert S_0' which is the stock price minus the present value of the dividends, for the stock price S_0. If the underlying is a currency, then portfolio A is adjusted to start off with $(1 + \rho)^{-T}$ units of the currency. This will grow to one unit at T, and the payoffs will be the same as in Table 3.11. Then put-call parity for currency options will become

$$S_0(1 + \rho)^{-T} + P_e(S_0,T,X) = C_e(S_0,T,X) + X(1 + r)^{-T}.$$

While put-call parity is an extremely important and useful result, it does not hold so neatly if the options are American. The put-call parity rule for American options must be stated as inequalities,

$$C_a(S_0', T, X) + X + \sum_{j=1}^{N} D_j(1 + r)^{-t_j} \geq S_0 + P_a(S_0', T, X)$$
$$\geq C_a(S_0', T, X) + X(1 + r)^{-T},$$

where S_0' is, again, the stock price minus the present value of the dividends. Although we shall skip the formal proof of this statement, it is, nonetheless, easy to make the calculations to determine if the rule is violated.

Now let us take a look at whether put-call parity holds for the DCRB options. Since the stock does not pay dividends, the DCRB calls are effectively European calls. The puts, however, are strictly

Table 3.12 Put-Call Parity for DCRB Calls

A European Put-Call Parity

Top row of cell: $S_0 + P_e(S_0,T,X)$
Bottom row of cell: $C_e(S_0,T,X) + X(1 + r)^{-T}$

Exercise Price	May	June	July
120	128.69	135.19	139.59
	128.6472	134.8880	139.9662
125	130.54	137.44	142.54
	130.6429	137.9667	142.6273
130	133.29	140.19	145.59
	133.4886	140.7953	145.3884

B American Put-Call Parity

Top row of cell: $C_a(S_0,T,X) + X$ (note: no dividends so $S_0' = S_0$)
Middle row of cell: $S_0 + P_a(S_0,T,X)$
Bottom row of cell: $C_a(S_0,T,X) + X(1 + r)^{-T}$

Exercise Price	May	June	July
120	128.75	135.40	140.90
	128.69	135.19	139.59
	128.6472	134.8880	139.9662
125	130.75	138.50	143.60
	130.54	137.44	142.54
	130.6429	137.9667	142.6273
130	133.60	141.35	146.40
	133.29	140.19	145.59
	133.4886	140.7953	145.3884

Note: Risk-free rates are 4.57% (May), 4.56% (June), and 4.63% (July); times to expiration are 0.0192 (May), 0.0959 (June), and 0.1726 (July).

American. But first let us apply the European put-call parity rule. Panel A of Table 3.12 shows the appropriate calculations. Each cell represents a combination of exercise price and expiration and contains two figures. The upper figure is the value of the stock plus the put, while the lower is the value of the call plus the bonds. If put-call parity holds, these two figures should be identical. Of course, we might not expect it to hold perfectly, because transaction costs may prevent some arbitrage opportunities from being worth exploiting. In most of the cases, the figures are very close.

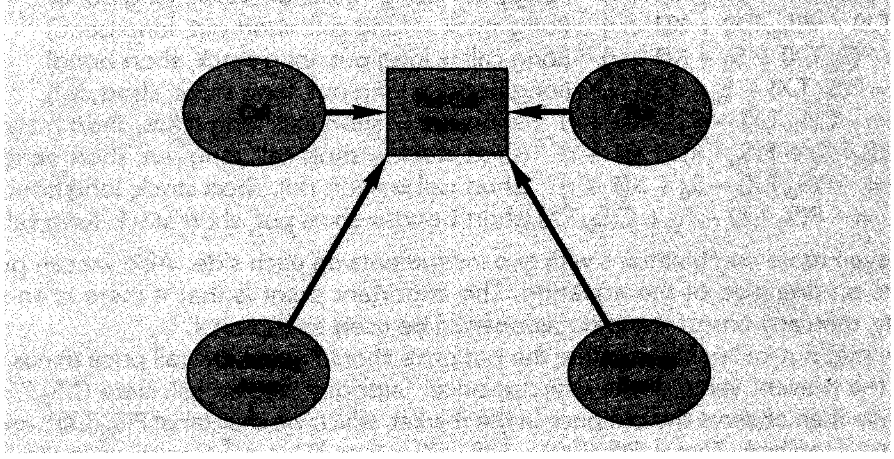
Now we apply the American put-call parity rule. Panel B contains three figures for each cell. Recalling that there are no dividends, the top figure is $C_a(S_0,T,X) + X$, the middle figure is $S_0 + P_a(S_0,T,X)$, and the bottom figure is $C_a(S_0,T,X) + X(1 + r)^{-T}$. These figures should line up in descending order—that is, the top figure should be no lower than the middle figure, which should be no lower than the bottom figure in each cell. This is the case only for the May and June 120 options, though the discrepancies might not be large enough to justify an arbitrage transaction.

Put-call parity is the linkage between the call, put, underlying asset, and risk-free bond. Figure 3.11 illustrates these relationships. Throughout the book, we shall further develop this figure to show other important relationships between derivative contracts, the underlying asset, and the risk-free bond market.

Effect of Interest Rates

Interest rates affect a put option in several ways. First, they affect the lower bound. The lower the interest rate, the higher the lower bound, reflecting the higher present value of the exercise price. In the extreme case of a zero interest rate, the lower bound is the same as the intrinsic value. Nonetheless, the put price will remain above the lower bound, because there is nothing special about a put if the interest rate is zero. But there is another effect of interest rates on put prices. When you finally sell the stock by exercising the put, you receive X dollars. If interest rates are higher, the X dollars will have a lower present value. Thus, a put holder forgoes higher interest while waiting to exercise the option and receive the exercise price. So higher interest rates make puts less attractive to investors.

FIGURE 3.11 The Linkage between Calls, Puts, and Underlying Asset and Risk-Free Bonds



Effect of Stock Volatility

The effect of volatility on a put's price is the same as that for a call: Higher volatility increases the possible gains for a put holder. For example, in our discussion of the effect of volatility on a call, we considered four equally likely stock prices at expiration for DCRB: 110, 120, 130, and 140. The four possible put prices at expiration for a 125 put are 15, 5, 0, and 0. If the volatility increases so that the four possible stock prices at expiration are 100, 120, 130, and 150, the four possible put prices at expiration are

25, 5, 0, and 0. For the holder of a put, this increase in volatility is desirable because the put price now can rise much higher. It does not matter that the put can be even deeper out-of-the money when it expires, because its lowest possible value is still zero. The put holder's loss thus is truncated. Therefore, the put will have a higher price if the volatility is higher.

Another approach to understanding the volatility effect is to consider a European put on a stock with zero volatility. If the put is currently in-the-money, it will be worth the present value of X minus the stock price S_0, because no further changes in the stock price other than the risk-free return will be expected. If the put is out-of-the-money, it will be worthless, because it will have no chance of expiring in-the-money. Either of these cases would be like a risk-free asset and would have no use for either hedgers or speculators. In addition, the lower bound and put price would remain below the intrinsic value, reflecting the fact that a European put still cannot be exercised until expiration.

As we discussed in the section on calls, we shall take up the role and measurement of volatility in Chapters 4 and 5.

DERIVATIVES TOOLS

Concepts, Applications, and Extensions
Put-Call Parity Arbitrage

The put-call parity equation states that

$$P(S_0,T,X) + S_0 = C(S_0,T,X) + X(1 + r)^{-T}.$$

The interpretation of the equation is that the left-hand side is a long put and long stock and the right-hand side is a long call and long risk-free bond. Positive signs refer to long positions and negative signs refer to short positions. There are a numerous other ways in which put-call parity can be rewritten:

$P(S_0,T,X) = C(S_0,T,X) - S_0 + X(1 + r)^{-T}$ (long put = long call, short stock, long bond)
$S_0 = C(S_0,T,X) - P(S_0,T,X) + X(1 + r)^{-T}$ (long stock = long call, short put, long bond)
$C(S_0,T,X) = P(S_0,T,X) + S_0 - X(1 + r)^{-T}$ (long call = long put, long stock, short bond)
$X(1 + r)^{-T} = P(S_0,T,X) + S_0 - C(S_0,T,X)$ (long bond = long put, long stock, short call)
$-P(S_0,T,X) = -C(S_0,T,X) + S_0 - X(1 + r)^{-T}$ (short put = short call, long stock, short bond)
$-S_0 = -C(S_0,T,X) + P(S_0,T,X) - X(1 + r)^{-T}$ (short stock = short call, long put, short bond)
$-C(S_0,T,X) = -P(S_0,T,X) - S_0 + X(1 + r)^{-T}$ (short call = short put, short stock, long bond)
$-X(1 + r)^{-T} = -P(S_0,T,X) - S_0 + C(S_0,T,X)$ (short bond = short put, short stock, long call)

There are even more combinations with two instruments on each side. Also we can put all four instruments on one side of the equation. The important point is that if there is an arbitrage opportunity, then any one of these equations can be used to exploit it.

For example, put-call parity says that the put price should equal the call price minus the stock price plus the present value of the exercise price. Suppose that we calculate $C(S_0,T,X) - S_0 + X(1 + r)^{-T}$. We then observe the put price in the market, which we shall label $P(S_0,T,X)^*$, and we find that $P(S_0,T,X)^*$ is too high. That is, $P(S_0,T,X)^* > C(S_0,T,X) - S_0 + X(1 + r)^{-T}$. So now what should we do?

Intuition says that the put price is too high so we should sell it. The first equation in the list above says that a put is equivalent to a combination of a long call, short stock, and long bond. Thus, this combination is equivalent to a put and is sometimes called a synthetic put. So if we sell the actual put and buy the synthetic put, we should be hedged, and yet we receive more from the sale of the actual put than we paid for the synthetic put. Here are the outcomes:

Instrument	Value at Expiration $S_T \le X$	$S_T > X$
Short put	$-(X - S_T)$	0
Long call	0	$S_T - X$
Short stock	$-S_T$	$-S_T$
Long bond	X	X
Total	0	0

The absence of any positive or negative payoff means that this is a perfect hedge. But in fact, we could detect this mispricing using any of the above equations. Let us pick the second one, $S_0 = C(S_0,T,X) - P(S_0,T,X) + X(1 + r)^{-T}$. We use the market price of the put for $P(S_0,T,X)$ on the right-hand side. Since we know that $P(S_0,T,X)^*$ is too high, we will be subtracting too much from the right-hand side. Then the right-hand side will be too low relative to the left-hand side. So, we should sell the left-hand side (meaning to sell short the stock) and buy the right-hand side (buy the call, sell the put, and buy the bond). Indeed, this is the strategy illustrated above.

It is even more important to see that it does not matter if the put is the mispriced instrument. We do not need to actually identify whether the put, call, or stock is mispriced. In the above transaction, we see that we are short the put, long the call, and short the stock. This means that the same transaction would be done if the put were overpriced or the call were underpriced or the stock were overpriced. We do not need to determine which instrument is the one mispriced. We simply plug into the formula and the equation will tell us what to do. All we need to do is remember to always sell overpriced instruments and buy underpriced instruments.

Questions and Problems

1. Suppose that you observe a European call option that is priced at less than the value $\text{Max}[0, S_0 - X(1 + r)^{-T}]$. What type of transaction should you execute to achieve the maximum benefit? Demonstrate that your strategy is correct by constructing a payoff table showing the outcomes of expiration.
2. Critique the following statement, made by an options investor: "My call option is very deep-in-the-money. I don't see how it can go any higher. I think I should exercise it."
3. Explain why an option's time value is greatest when the stock price is near the exercise price and why it nearly disappears when the option is deep-in- or out-of-the-money.
4. What would happen in the options market if the price of an American call were less than the value $\text{Max}(0, S_0 - X)$? Would your answer differ if the option were European?
5. Consider an option that expires in 68 days. The bid and ask discounts on the Treasury bill maturing in 67 days are 8.20 and 8.24, respectively. Find the approximate risk-free rate.
6. Why do higher interest rates lead to higher call option prices but lower put option prices?
7. The value $\text{Max}[0, X(1 + r)^{-T} - S_0]$ was shown to be the lowest possible value of a European put. Why is this value irrelevant for an American put?
8. Call prices are directly related to the stock's volatility, yet higher volatility means that the stock price can go lower. How would you resolve this apparent paradox?
9. Why does the justification for exercising an American call early not hold up when considering an American put?
10. In this chapter, we did not learn how to obtain the exact price of a call without knowing the price of the put and using put-call parity. In one special case, however, we can obtain an exact price for a call. Assume that the option has an infinite maturity. Then use the maximum and minimum values we learned in this chapter to obtain the prices of European and American calls.

11. Why might two calls or puts alike in all respects but time to expiration have approximately the same price?
12. Why might two calls or puts alike in all respects but exercise price have approximately the same price?
13. Suppose a European put price exceeds the value predicted by put-call parity. How could an investor profit? Demonstrate that your strategy is correct by constructing a payoff table showing the outcomes at expiration.

The following option prices were observed for a stock on July 6 of a particular year. Use this information to solve problems 14 through 19. Unless otherwise indicated, ignore dividends on the stock. The stock is priced at 165.13. The expirations are July 17, August 21, and October 16. The risk-free rates are 0.0516, 0.0550, and 0.0588, respectively.

	Calls			Puts		
Strike	**Jul**	**Aug**	**Oct**	**Jul**	**Aug**	**Oct**
155	10.50	11.80	14.00	0.20	1.25	2.75
160	6.00	8.10	11.10	0.75	2.75	4.50
165	2.70	5.20	8.10	2.35	4.70	6.70
170	0.80	3.20	6.00	5.80	7.50	9.00

14. Examine the following pairs of puts, which differ only by exercise price. Determine if any violate the rules regarding relationships between American options that differ only by exercise price.
    a. August 155 and 160
    b. October 160 and 170
15. Examine the following pairs of calls, which differ only by exercise price. Determine whether any violate the rules regarding relationships between American options that differ only by exercise price.
    a. August 155 and 160
    b. October 160 and 165
16. Compute the intrinsic values, time values, and lower bounds of the following calls. Identify any profit opportunities that may exist. Treat these as American options for purposes of determining the intrinsic values and time values and European options for the purpose of determining the lower bounds.
    a. July 160
    b. October 155
    c. August 170
17. Compute the intrinsic values, time values, and lower bounds of the following puts. Identify any profit opportunities that may exist. Treat these as American options for purposes of determining the intrinsic values and time values and as European options for the purpose of determining the lower bounds.
    a. July 165
    b. August 160
    c. October 170
18. Check the following combinations of puts and calls, and determine whether they conform to the put-call parity rule for European options. If you see any violations, suggest a strategy.
    a. July 155
    b. August 160
    c. October 170
19. Repeat Question 18 using American put-call parity, but do not suggest a strategy.

20. Suppose that the current stock price is \$100, the exercise price is \$100, the annually compounded interest rate is 5 percent, the stock pays a \$1 dividend in the next instant, and the quoted call price is \$3.50 for a one year option. Identify the appropriate arbitrage opportunity and show the appropriate arbitrage strategy.
21. (Concept Question) Suppose Congress decides that investors should not profit when stock prices go down so it outlaws short selling. Congress has not figured out options, however, so there are no restrictions on option trading. Explain how to accomplish the equivalent of a short sale by using options.
22. (Concept Problem) Put-call parity is a powerful formula that can be used to create equivalent combinations of options, risk-free bonds, and stock. Suppose that there are options available on the number of points Shaquille O'Neal will score in his next game. For example, a call option with an exercise price of 32 would pay off Max(0, S_0 – 32), where S_0 is the number of points Shaq has recorded by the end of the game. Thus, if he scores 35, call holders receive \$3 for each call. If he scores less than 32, call holders receive nothing. A put with an exercise price of 32 would pay off Max(0, 32 – S_0). If Shaq scores more than 32, put holders receive nothing. If he scores 28, put holders receive \$4 for each put. Obviously there is no way to actually buy a position in the underlying asset, a point. However, put-call parity shows that the underlying asset can be recreated from a combination of puts, calls, and risk-free bonds. Show how this would be done, and give the formula for the price of a point.
23. Suppose that the current stock price is \$90, the exercise price is \$100, the annually compounded interest rate is 5 percent, the stock pays a \$1 dividend in the next instant, and the quoted put price is \$6 for a one year option. Identify the appropriate arbitrage opportunity and show the appropriate arbitrage strategy.
24. On December 9 of a particular year, a January Swiss franc call option with an exercise price of 46 had a price of 1.63. The January 46 put was at 0.14. The spot rate was 47.28. All prices are in cents per Swiss franc. The option expired on January 13. The U.S. risk-free rate was 7.1 percent, while the Swiss risk-free rate was 3.6 percent. Do the following:
    a. Determine the intrinsic value of the call.
    b. Determine the lower bound of the call.
    c. Determine the time value of the call.
    d. Determine the intrinsic value of the put.
    e. Determine the lower bound of the put.
    f. Determine the time value of the put.
    g. Determine whether put-call parity holds.

Appendix 3

The Dynamics of Option Boundary Conditions: A Learning Exercise

As we have seen in this chapter, option prices are limited by various boundaries. We examined maximum prices and minimum prices, the latter of which can differ depending on whether the option is European or American. We also alluded to the fact that these boundaries change over time. This book comes with a Microsoft Excel® spreadsheet called BoundaryConditions7e.xls, which can be used to observe how these boundary conditions, as well as the option price, change with changes in various inputs.

To use the spreadsheet, you will need Windows 95 or higher. The spreadsheet BoundaryConditions7e.xls is available as a download via the product support Web site. To access it:

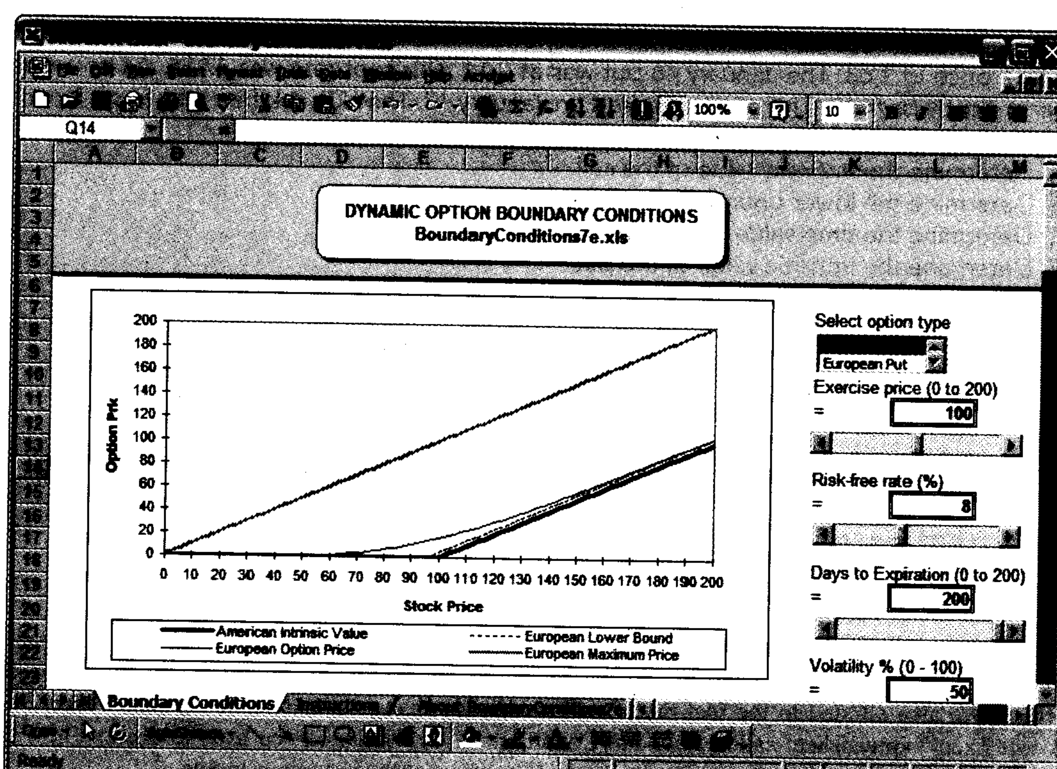
1. Go to www.academic.cengage.com/aise.
2. Click on Instructor Resources or Student Resources.
3. Locate the title of this book on the Web page.
4. Download and install the spreadsheet using the link provided.

Download and access the spreadsheet to observe the sample case, a European call with exercise price of 100, risk-free rate of 8 percent, 200 days until expiration, and a volatility of 50. At this point, we have not specifically studied how the volatility is measured. We mentioned that volatility is usually interpreted as the standard deviation of the stock return. In Chapters 4 and 5, we shall get into a more specific treatment of volatility. For this exercise, you should just be prepared to enter a value between 0 and 100 (%) for the volatility. Excel computes the option price based on the model that we shall cover in Chapter 5.

Observe the graph, which shows four lines: the maximum price of a European call, the lower bound for a European call, the European option price, and the intrinsic value of an American call. Each input value is inserted into a cell. You can enter any reasonable value, but the graph will print the results for a range of stock prices from 0 to 200, so you should not choose an exercise price more than 200.

To observe how these lines change, you can slide the scroll bars left or right, which will automatically change the appropriate input value. Alternatively, you can click on the arrows at each end of the scroll bar for automatic incrementing of the appropriate input value. If you choose to slide the scroll bar, the graph will be redrawn only when you release the left mouse button. By dragging the scroll bar, you can observe the graph change.

Do the following exercise. You will be asked several questions. References to locations in the chapter where the answers can be found are provided.



- Select European Calls
- Drag the Days to Expiration from 200 gradually down to zero. What happens?
  1. The call price converges to the lower bound. Do you know why?
  2. The lower bound converges downward to the intrinsic value. Do you know why?
- (With Days to Expiration at 200) Drag the volatility from 50 percent down to 0 percent. What happens?
  1. The call price converges to the lower bound. Do you know why?
  2. The lower bound remains above the American intrinsic value. Do you know why?
- (With Days to Expiration at 200 and Volatility at 50 percent) Drag the risk-free rate from 8 percent down to zero. What happens?
  1. The lower bound converges downward to the intrinsic value. Do you know why?
  2. The call price stays above the lower bound. Do you know why?

Now do the same exercise for puts

- Select European puts
- Drag the Days to Expiration from 200 gradually down to zero. What happens?
  1. The put price converges to the lower bound. Do you know why?
  2. The lower bound converges upward to the intrinsic value. Do you know why?
- (With Days to Expiration at 200) Drag the volatility from 50 percent down to 0 percent. What happens?
  1. The put price converges to the lower bound. Do you know why?
  2. The lower bound remains below the American intrinsic value. Do you know why?
- (With Days to Expiration at 200 and Volatility at 50 percent) Drag the risk-free rate from 8 percent down to zero. What happens?
  1. The lower bound converges upward to the intrinsic value. Do you know why?
  2. The put price stays above the lower bound. Do you know why?

4

CHAPTER

OPTION PRICING MODELS: THE BINOMIAL MODEL

This chapter examines the first of two general types of option pricing models. A model is a simplified representation of reality that uses certain inputs to produce an output, or result. An option pricing model is a mathematical formula or computational procedure that uses the factors determining the option's price as inputs. The output is the theoretical fair value of the option. If the model performs as it should, the option's market price will equal the theoretical fair value. Obtaining the theoretical fair value is a process called option pricing.

In Chapter 3, we examined some basic concepts in determining option prices. We saw, however, only how to price options relative to other options; for example, put-call parity demonstrates that given the price of a call, one can determine the price of a put. We also discovered relationships among the prices of options that differ by exercise prices and examined the upper and lower bounds on call and put prices. We did not, however, learn how to determine the exact option price directly from the factors that influence it. A large body of research on option pricing exists. Much of it goes far beyond the intended level of this book. The models range from the relatively simple to the extremely complex. All of the models have much in common, but it is necessary to understand the basic models before moving on to the more complex but more realistic ones.

We begin with a simple model called the binomial option pricing model, which is more of a computational procedure than a formula. After taking the binomial model through several stages, we move on to Chapter 5 and the Black-Scholes-Merton option pricing model, which is a mathematical formula. In both cases, however, we have the same objective: to obtain the theoretical fair value of the option, which is the price at which it should be trading.

ONE-PERIOD BINOMIAL MODEL

First, let us consider what we mean by a one-period world. An option has a defined life, typically expressed in days. Assume that the option's life is one unit of time. This time period can be as short or as long as necessary. If the time period is one day and the option has more than one day remaining, we will need a multiperiod model, which we shall examine later. For now, we will assume that the option's life is a single time period.

The model is called a binomial model. It allows the stock price to go either up or down, possibly at different rates. A binomial probability distribution is a distribution in which there are two outcomes or states. The probability of an up or down movement is governed by the binomial probability distribution. Because of this, the model is also called a two-state model.

In applying the binomial model to an underlying asset, however, it is immediately obvious that the range of possible outcomes is greater than the two states that the binomial distribution can accommodate; however, that makes the model no less worthwhile. Its virtues are its simplicity and its ability to present the fundamental concepts of option pricing models clearly and concisely. In so doing, it establishes a foundation that facilitates an understanding of the Black-Scholes-Merton model.

Consider a world in which there is a stock priced at S on which call options are available.[1] The call has one period remaining before it expires. The beginning of the period is today and is referred to as time 0. The end of the period is referred to as time 1. When the call expires, the stock can take on one of two values: It can go up by a factor of u or down by a factor of d. If it goes up, the stock price will be uS. If it goes down, it will be dS.

For example, suppose that the stock price is currently $50 and can go either up by 10 percent or down by 8 percent. Thus, $u = 1.10$ and $d = 1 - 0.08 = 0.92$. The variables u and d, therefore, are 1.0 plus the rates of return on the stock. When the call expires, the stock will be either $50(1.10) = 55$ or $50(0.92) = 46$.

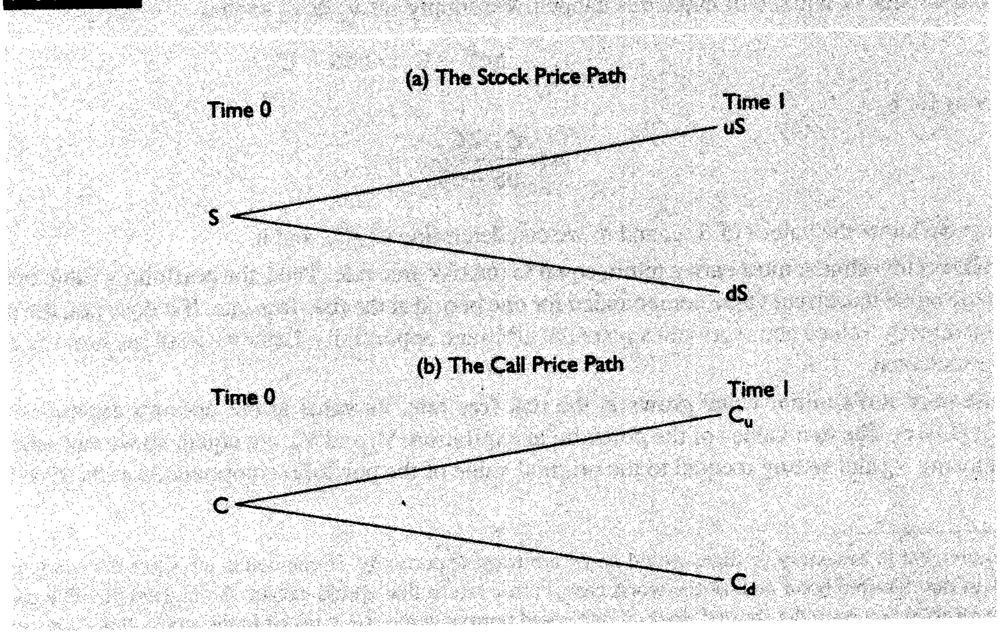
Consider a call option on the stock with an exercise price of X and a current price of C. When the option expires, it will be worth either C_u or C_d. Because at expiration the call price is its intrinsic value, then

$$C_u = \text{Max}[0, uS - X]$$
$$C_d = \text{Max}[0, dS - X].$$

Figure 4.1 illustrates the paths of both the stock and the call prices. This diagram is simple, but it will become more complex when we introduce the two-period model.

If both stock prices resulted in the option expiring in-the-money, the option would not be very speculative; however, it would still be correctly priced by the model. The writer would receive a premium compensating for the future cash outflow expected upon exercising the option. To make things more

FIGURE 4.1 The One-Period Binomial Tree



[1]The model can also price put options. We shall see how this is done in a later section.

interesting, however, we shall define our variables such that the option has a chance of expiring out-of-the-money. Assume that dS is less than X; that is, if the stock price goes down, the option will expire out-of-the-money. Also assume that uS is greater than X such that if the stock price goes up, the option will expire in-the-money.

Let the per period risk-free rate be identified by the symbol r. The risk-free rate is the interest earned on a riskless investment over a time period equal to the option's remaining life. The risk-free rate is between the rate of return if the stock goes up and the rate of return if the stock goes down. Thus, $d < 1 + r < u$.[2] We shall assume that all investors can borrow or lend at the risk-free rate.

The objective of this model is to derive a formula for the theoretical fair value of the option, the variable C. The theoretical fair value is then compared to the actual price and reveals whether the option is overpriced, underpriced, or correctly priced. The formula for C is developed by constructing a riskless portfolio of stock and options. A riskless portfolio should earn the risk-free rate. Given the stock's values and the riskless return on the portfolio, the call's value can be inferred from the other variables.

This riskless portfolio is called a hedge portfolio and consists of h shares of stock and a single written call. The model provides the hedge ratio, h. The current value of the portfolio is the value of the h shares minus the value of the short call. We subtract the call's value from the value of the h shares because the shares are assets and the short call is a liability. Thus, the portfolio value is assets minus liabilities, or simply net worth. The current portfolio value is denoted as V, where $V = hS - C$. Alternatively, we can think of this portfolio as requiring that we purchase h shares at S per share, but we offset some of this price by selling one call for C. Thus, $V = hS - C$ is the amount of our own money required to construct this portfolio.

At expiration, the portfolio value will be either V_u if the stock goes up or V_d if the stock goes down. Using the previously defined terms,

$$V_u = huS - C_u$$
$$V_d = hdS - C_d.$$

Think of V_u and V_d as the amount of money we can obtain by liquidating the portfolio when the option expires. If the same outcome is achieved regardless of what the stock price does, the position is riskless. We can choose a value of h that will make this happen. We simply set $V_u = V_d$ so that

$$huS - C_u = hdS - C_d.$$

Solving for h,

$$h = \frac{C_u - C_d}{uS - dS}.$$

Since we know the values of S, u, and d, we can determine C_u, C_d, and h.

A riskless investment must earn a return equal to the risk-free rate. Thus, the portfolio's value one period later should equal its current value compounded for one period at the risk-free rate. If it does not, the portfolio will be incorrectly valued and represent a potential arbitrage opportunity. Later we shall see how the arbitrage would be executed.

If the portfolio's initial value grows at the risk-free rate, its value at the option's expiration will be $(hS - C)(1 + r)$. The two values of the portfolio at expiration, V_u and V_d, are equal, so we can select either one. Choosing V_u and setting it equal to the original value of the portfolio compounded at the risk-free rate gives

[2]This requirement is necessary or there would be an arbitrage opportunity. If one could sell short the stock, invest the proceeds in the risk-free bond and, in the worst case, earn a return that would exceed the highest possible payout that would be owed to buy back the shorted stock. If one could borrow at the rate r, invest in the stock, and always earn more than the cost of the loan, then one could make unlimited amounts of money without committing any funds.

$$V(1 + r) = V_u,$$
$$(hS - C)(1 + r) = huS - C_u.$$

Substituting the formula for h and solving this equation for C gives the option pricing formula,

$$C = \frac{pC_u + (1-p)C_d}{1+r},$$

where p is defined as $(1 + r - d)/(u - d)$.

The formula gives the call option price as a function of the variables C_u, C_d, p, and r; however, C_u and C_d are determined by the variables S, u, d, and X. Thus, the variables affecting the call option price are the current stock price, S, the exercise price, X, the risk-free rate, r, and the parameters, u and d, which define the possible future stock prices at expiration. Notice how the call price is a weighted average of the two possible call prices the next period, discounted at the risk-free rate. Notice also that we never specified the probabilities of the two stock price movements; they do not enter into the model. The option is priced relative to the stock. Therefore, given the stock price, one can obtain the option price. The stock, however, is priced independently of the option, and thus the probabilities of the stock price movements would be a factor in pricing the stock. But pricing the stock is not our concern. We already have the stock price, S.

In Chapter 1, we introduced the concept of risk neutrality and noted that we would be using it to price derivatives. The investors' feelings about risk play an important role in the pricing of securities, but in the risk-neutral option pricing framework, investors' sensitivities to risk are of no consequence. This does not mean, however, that the model assumes that investors are risk neutral. The stock price is determined by how investors feel about risk. If investors are risk neutral and determine that a stock is worth \$20, the model will use \$20 and take no account of investors' feelings about risk. If investors are risk averse and determine that a stock is worth \$20, the model will use \$20 and disregard investors' feelings about risk. This does not mean that the stock will be priced equally by risk-averse and risk-neutral investors; rather, the model will accept the stock price as given and pay no attention to how risk was used to obtain the stock price. In other words, given the stock price, both an aggressive and a conservative investor will assign the same price to an option on the stock.

Illustrative Example

Consider a stock currently priced at \$100. One period later it can go up to \$125, an increase of 25 percent, or down to \$80, a decrease of 20 percent. Assume a call option with an exercise price of \$100. The risk-free rate is 7 percent. The inputs are summarized as follows:

$$S = 100 \qquad d = 0.80$$
$$X = 100 \qquad r = 0.07$$
$$u = 1.25$$

First, we find the values of C_u and C_d:

$$\begin{aligned} C_u &= \text{Max}[0, uS - X] \\ &= \text{Max}[0, 100(1.25) - 100] \\ &= 25 \\ C_d &= \text{Max}[0, dS - X] \\ &= \text{Max}[0, 100(0.80) - 100] \\ &= 0. \end{aligned}$$

The hedge ratio, h, is

$$h = \frac{25 - 0}{125 - 80} = 0.556.$$

The hedge requires 0.556 shares of stock for each call.[3] The value of p is

$$p = \frac{1 + r - d}{u - d} = \frac{1.07 - 0.80}{1.25 - 0.80} = 0.6.$$

Then

$$1 - p = 1 - 0.6 = 0.4.$$

Plugging into the formula for C gives

$$C = \frac{(0.6)25 + (0.4)0}{1.07} = 14.02$$

Thus, the theoretical fair value of the call is $14.02.

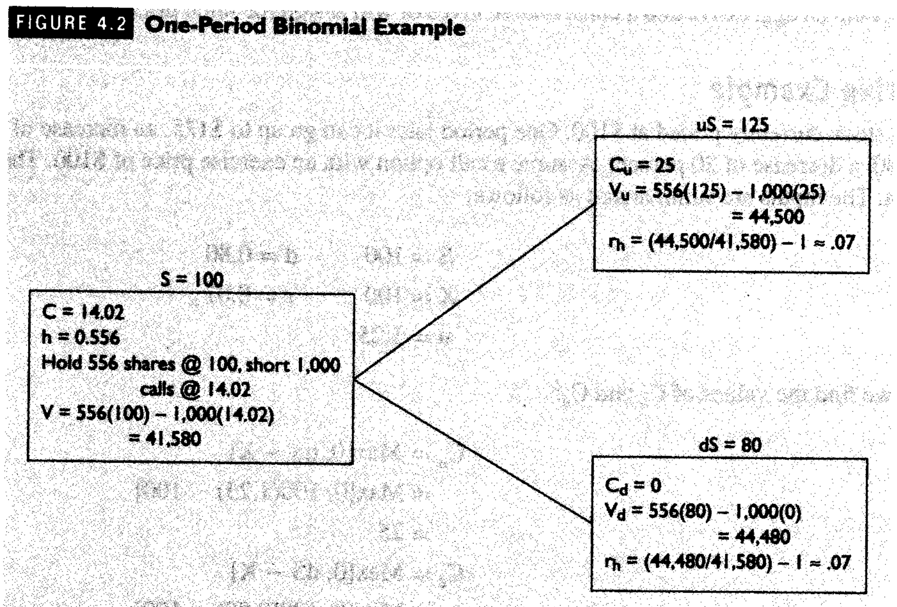
Hedge Portfolio

Consider a hedge portfolio consisting of a short position in 1,000 calls and a long position in 556 shares of stock. The number of shares is determined by the hedge ratio of 0.556 shares per written call. The current value of this portfolio is

$$556(\$100) - 1{,}000(\$14.02) = \$41{,}580.$$

Thus, the investor buys 556 shares at $100 per share and writes 1,000 calls at $14.02. This requires a payment of 556($100) = $55,600 for the stock and takes in for the calls. The net cash outlay is $55,600 − $14,020 = $41,580. This total represents the assets (the stock) minus the liabilities (the calls), and thus is the net worth, or the amount the investor must commit to the transaction. Figure 4.2 illustrates the process.

FIGURE 4.2 One-Period Binomial Example



[3] We assume that odd lots of stock can be purchased, but we do not permit the purchase of fractional shares.

If the stock goes up to \$125, the call will be exercised for a value of \$125 − \$100 = \$25. The stock will be worth 556(\$125) = \$69,500. Thus, the portfolio will be worth 556(\$125) − 1,000(\$125 − \$100) = \$44,500. If the stock goes down to \$80, the call will expire out-of-the-money. The portfolio will be worth 556(\$80) = \$44,480. These two values of the portfolio at expiration are essentially equal, because the \$20 difference is due only to the rounding off of the hedge ratio. The return on this hedge portfolio is

$$r_h = \left(\frac{\$44,500}{\$41,580}\right) - 1 \approx 0.07$$

which is the risk-free rate. The original investment of \$41,580 will have grown to \$44,500—a return of about 7 percent, the risk-free rate.

If the call price were not \$14.02, an arbitrage opportunity would exist. First we will consider the case where the call is overpriced.

DERIVATIVES TOOLS

Concepts, Applications, and Extensions
Binomial Option Pricing, Risk Premiums, and Probabilities

You have observed that the binomial model does not require the actual probabilities of up and down moves. But what if we had the actual probabilities of the up and down moves? Would our binomial tree be inconsistent with this information? Let us take a look.

Go back to our example of the stock at 100 with u = 1.25, d = 0.80, and r = 0.07. Suppose we knew that the probability of an up move is 0.7 and the probability of a down move is 0.3. That means that the expected stock price at time 1 would be

$$(0.7)125 + (0.3)80 = 111.50$$

If the current stock price is 100, then investors must be discounting the expected stock price at a rate of 11.5 percent. That is, 111.50/1.115 = 100. Thus, 11.5 percent is the required return on the stock. Since the risk-free rate is 7 percent, investors are requiring a risk premium of 4.5 percent for this stock.

Given that we know something about how investors value the stock, can we use that information to value the call option? Remember that we found that a one-period call with an exercise price of 100 is worth 14.02. The payoffs at time 1 were 25 and 0. Thus, the expected option price is

$$(0.7)25 + (0.3)0 = 17.50.$$

If we apply the same discount rate to the option as we did to the stock, we would obtain a current option price of

$$\frac{17.50}{1.115} = 15.70$$

But if 15.70 is the price of the call option, we can easily earn an arbitrage profit. We demonstrated that this option, which should be worth 14.02, could be used to generate a riskless return of 9.6 percent if it is selling at 15. Hence, we could earn even more if the call were selling for 15.70. Clearly something is wrong.

What is wrong is that the required return on the option cannot be the same as that of the stock. To be priced at 14.02, the required return must be

$$\frac{17.50}{14.02} - 1 = 0.248.$$

In other words, the required return on the option must be almost 25 percent, which is more than twice that of the stock.

Options are riskier than stock and, consequently, they must have higher required rates of return. We can determine how the required return on the call relates to the required return on the stock by using the binomial model.

Remember that if the stock goes up or down, the return on the portfolio is the risk-free rate. So in either case, we have

$$\frac{V_u}{V} - 1 = \frac{huS - C_u}{hS - C} - 1 = r \text{ and}$$

$$\frac{V_d}{V} - 1 = \frac{huS - C_d}{hS - C} - 1 = r$$

The returns on the stock for the two outcomes are defined as

$$\frac{uS}{S} - 1 = r_s^u \quad \text{and} \quad \frac{dS}{S} - 1 = r_s^d$$

The returns on the call for the two outcomes are defined as

$$\frac{C_u}{C} - 1 = r_s^u \quad \text{and} \quad \frac{C_d}{C} - 1 = r_c^d$$

We shall need these in the form of $C(1 + r_c^u) = C_u$ and $C(1 + r_c^d) = C_d$. The initial value of the portfolio of $hS - C$ should grow at the risk-free rate to equal the value if the portfolio goes up of $huS - C_u$:

$$(1 + r)(hS - C) = huS - C_u.$$

Substituting $C(1 + r_c^u)$ for C_u and solving for r_c^u we obtain the equation

$$r_c^u = r + (r_s^u - r)h(S/C).$$

If we do the same for the case that the stock goes down. we obtain

$$r_c^d = r + (r_s^d - r)h(S/C).$$

By definition, the expected return on the call is a weighted average of the up and down returns, where the weights are the probabilities of the up and down returns, respectively:

$$E(r_c) = qr_c^u + (1 - q)r_c^d.$$

Substituting $r + (r_s^u - r)h(S/C)$ for r_c^u and $r + (r_s^d - r)h(S/C)$ for r_c^d and recognizing that $qr_s^u + (1 - q)r_s^d$ is the expected return on the stock, $E(r_s)$, we obtain the expected return on the call in terms of the expected return on the stock:

$$E(r_c) = r + [E(r_s) - r]h(S/C).$$

We see from this result that the expected return on the call is the risk-free rate plus a risk premium that is related to the risk premium on the stock, $E(r_s) - r$, and a factor, $h(S/C)$, that reflects the leverage on the call.

In our example, $h = 0.556$. Plugging in, we obtain

$$E(r_c) = 0.07 + [0.115 - 0.07]0.556(100/14.02) = 0.248.$$

If an asset is correctly priced in the market, its expected return equals the return required by investors. Thus, the required return on the option is the risk-free rate and a premium related to the stock's risk premium and the option's leverage. We see here that the required and expected returns are 24.8 percent, which we found would have to be the option's required return to justify a price of 14.02.

But regardless of what the expected return on the stock and call are or the true probabilities of the up and down moves, we can price the call using the arbitrage approach presented in this chapter and be confident that nothing we have done is incompatible with the expected returns and true probabilities.

Overpriced Call

If the call were overpriced, a riskless hedge could generate a riskless return in excess of the risk-free rate. Suppose the market price of the call is $15. If you buy 556 shares and write 1,000 calls, the value of the investment today is

$$556(\$100) - 1{,}000(\$15) = \$40{,}600.$$

If the stock goes up to $125, at expiration the call will be priced at $25 and the portfolio will be worth 556($125) − 1,000($25) = $44,500. If the stock goes down to $80, the call will be worth nothing and the portfolio will be worth 556($80) = $44,480. In either case, the portfolio will be worth the same, the difference of $20 again due to rounding. The initial investment of $40,600 will have grown to $44,500, a riskless return of

$$r_h = \left(\frac{\$44{,}500}{\$40{,}600}\right) - 1 \approx 0.096$$

which is considerably higher than the risk-free rate. In fact, the investor could have borrowed the $40,600 at the risk-free rate. Thus, the investor could have done this risk-free transaction without putting up any money.

A riskless portfolio that will earn more than the risk-free rate violates the law of one price. The risk-free bond and the hedge portfolio produce identical results but have different prices, $41,580 for the former and $40,600 for the latter. Obviously, this is a very attractive opportunity. All investors will recognize it and hurry to execute the transaction. This will increase the demand for the stock and the supply of the option. Consequently, the stock price will tend to increase and the option price to decrease until the option is correctly priced. For illustrative purposes, assume that the stock price stays at $100. Then the option price must fall from $15 to $14.02. Only at an option price of $14.02 will this risk-free portfolio offer a return equal to the risk-free rate.

Now consider what happens if the option is underpriced.

Underpriced Call

If the option is underpriced, it is necessary to buy it. To hedge a long option position, the investor must sell the stock short. Suppose the call is priced at $13. Then the investor sells short 556 shares at $100, which generates a cash inflow of 556($100) = $55,600. Now the investor buys 1,000 calls at $13 each for a cost of $13,000. This produces a net cash inflow of $42,600.

If the stock goes to $125, the investor buys it back at 556($125) = $69,500. He exercises the calls for a gain of 1,000($125 − $100) = $25,000. The net cash flow is −$69,500 + $25,000 = −$44,500. If the stock goes down to $80, the investor buys it back, paying 556($80) = $44,480 while the calls expire worthless. The $20 difference is again due to rounding.

In both outcomes, the returns are essentially equivalent. The overall transaction is like a loan in which the investor receives $42,600 up front and pays back $44,500 later. This is equivalent to an interest rate of ($44,500/$42,600) − 1 = 0.0446. Because this transaction is the same as borrowing at a rate of 4.46 percent and the risk-free rate is 7 percent, it is an attractive borrowing opportunity. The investor could have lent the original $42,600 at the risk-free rate, thereby earning a profit at no risk and with no commitments of funds. All investors will recognize this and execute the transaction. This will tend to drive up the call price, or possibly drive down the stock price, until equilibrium is reached. If the price of the stock stays at $100, equilibrium will be reached when the call price rises to $14.02.

This model considered only a single period. In the next section, we extend the model to a two-period world.

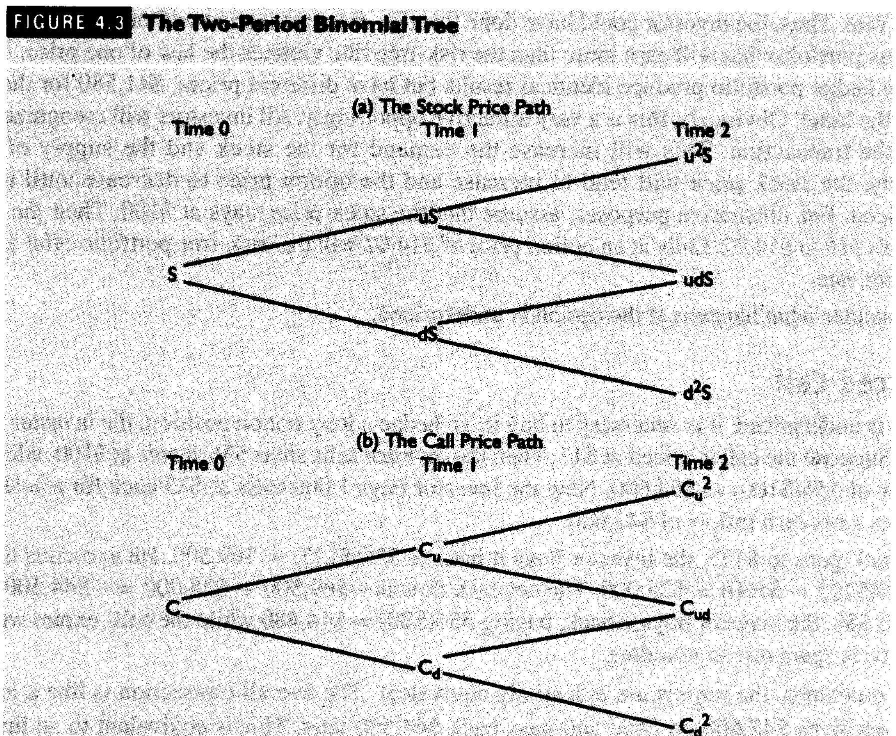
TWO-PERIOD BINOMIAL MODEL

In the single-period world, the stock price goes either up or down. Thus, there are only two possible future stock prices. To increase the degree of realism, we will now add another period. This will increase the number of possible outcomes at expiration. Thus, our model has three time points: today or time 0, time 1, and time 2.

Suppose that at the end of the first period the stock price has risen to uS. During the second period it could go either up or down, in which case it would end up at either u^2S or udS. If the stock price has gone down in the first period to dS, during the second period it will either go down again or go back up, in which case it will end up at either d^2S or duS. In this example, we let $d = 1/u$ which means that $udS = duS = S$. This is a convenient but not necessary assumption that is helpful when working with very basic versions of this model.

Figure 4.3 illustrates the paths of the stock price and the corresponding call prices. Viewed this way, we can see why the diagram is often called a binomial tree. The option prices at expiration are

FIGURE 4.3 The Two-Period Binomial Tree



$$C_{u^2} = \text{Max}\ [0, u^2S - X]$$

$$C_{ud} = \text{Max}\ [0, udS - X]$$

$$C_{d^2} = \text{Max}\ [0, d^2S - X]$$

The possible option prices at the end of the first period, C_u and C_d, initially are unknown; however, they can be found.

Suppose that in the first period the stock price increases to uS. Because there will be only one period remaining with two possible outcomes, the one-period binomial model is appropriate for finding the option price, C_u. If at the end of the first period the stock price decreases to dS, we will again find ourselves facing a single-period world with two possible outcomes. Here we can again use the one-period binomial model to obtain the value of C_d. Using the one-period model, the option prices C_u and C_d are

$$C_u = \frac{pC_{u^2} + (1-p)C_{ud}}{1+r}$$

and

$$C_d = \frac{pC_{ud} + (1-p)C_{d^2}}{1+r}$$

In a single-period world, a call option's value is a weighted average of the option's two possible values at the end of the next period. The call's value if the stock goes up in the next period is weighted by the factor p; its value if the stock goes down in the next period is weighted by the factor $1 - p$. To obtain the call price at the start of the period, we discount the weighted average of the two possible future call prices at the risk-free rate for one period. The single-period binomial model is, thus, a general formula that can be used in any multiperiod world when there is but one period remaining.

Even if the call does not expire at the end of the next period, we can use the formula to find the current call price, the theoretical fair value, as a weighted average of the two possible call prices in the next period; that is,

$$C_d = \frac{pC_u + (1-p)C_d}{1+r}.$$

First, we find the values of C_u and C_d; then we substitute these into the above formula for C.

For a more direct approach, we can use

$$C = \frac{p^2C_{u^2} + 2p(1-p)C_{ud} + (1-p)^2C_{d^2}}{(1+r)^2}$$

This formula illustrates that the call's value is a weighted average of its three possible values at expiration two periods later. The denominator, $(1 + r)^2$ discounts this figure back two periods to the present.

Notice that we have not actually derived the formula by constructing a hedge portfolio. This is possible, however, and in a later section we shall see how the hedge portfolio works. First, note that the hedge is constructed by initially holding h shares of stock for each call written. At the end of the first period, the stock price is either uS or dS. At that point, we must adjust the hedge ratio. If the stock is at uS, let the new hedge ratio be designated as h_u; if at dS, let the new ratio be h_d. The formulas for h_u and h_d are of the same general type as that of h in the single-period model. The numerator is the call's value if the stock goes up the next period minus the call's value if the stock goes down the next period. The denominator is the price of the stock if it goes up the next period minus its price if it goes down the next period. In equation form,

$$h = \frac{C_u - C_b}{uS - dS}, h_u = \frac{C_{u^2} - C_{ud}}{u^2S - udS}, h_d = \frac{C_{ud} - C_{d^2}}{duS - d^2S}.$$

Illustrative Example

Consider the example in a two-period world from the previous section. All input values remain the same. The possible stock prices at expiration are

$$
\begin{aligned}
u^2S &= 100(1.25)^2 \\
&= 156.25 \\
udS &= 100(1.25)(0.80) \\
&= 100 \\
d^2S &= 100(0.80)^2 \\
&= 64.
\end{aligned}
$$

The call prices at expiration are

$$
\begin{aligned}
C_{u^2} &= \text{Max}[0, u^2S - X] \\
&= \text{Max}(0, 156.25 - 100) \\
&= 56.25 \\
C_{ud} &= \text{Max}[0, udS - X] \\
&= \text{Max}(0, 100 - 100) \\
&= 0 \\
C_{d^2} &= \text{Max}[0, d^2S - X] \\
&= \text{Max}(0, 64 - 100) \\
&= 0.
\end{aligned}
$$

The value of p is the same, $(1 + r - d)/(u - d)$, regardless of the number of periods in the model.

We can find the call's value by either of the two methods discussed in the previous section. Let us first compute the values of C_u and C_d:

$$C_u = \frac{(0.6)56.25 + (0.4)0}{1.07} = 31.54$$

$$C_d = \frac{(0.6)0 + (0.4)0}{1.07} = 0$$

Note why the value of the call is $0.00 at time 1 when the stock is at 80: The call cannot expire in-the-money at time 2; therefore it must be worth nothing at time 1. The value of the call at time 0 is a weighted average of the two possible call values one period later:

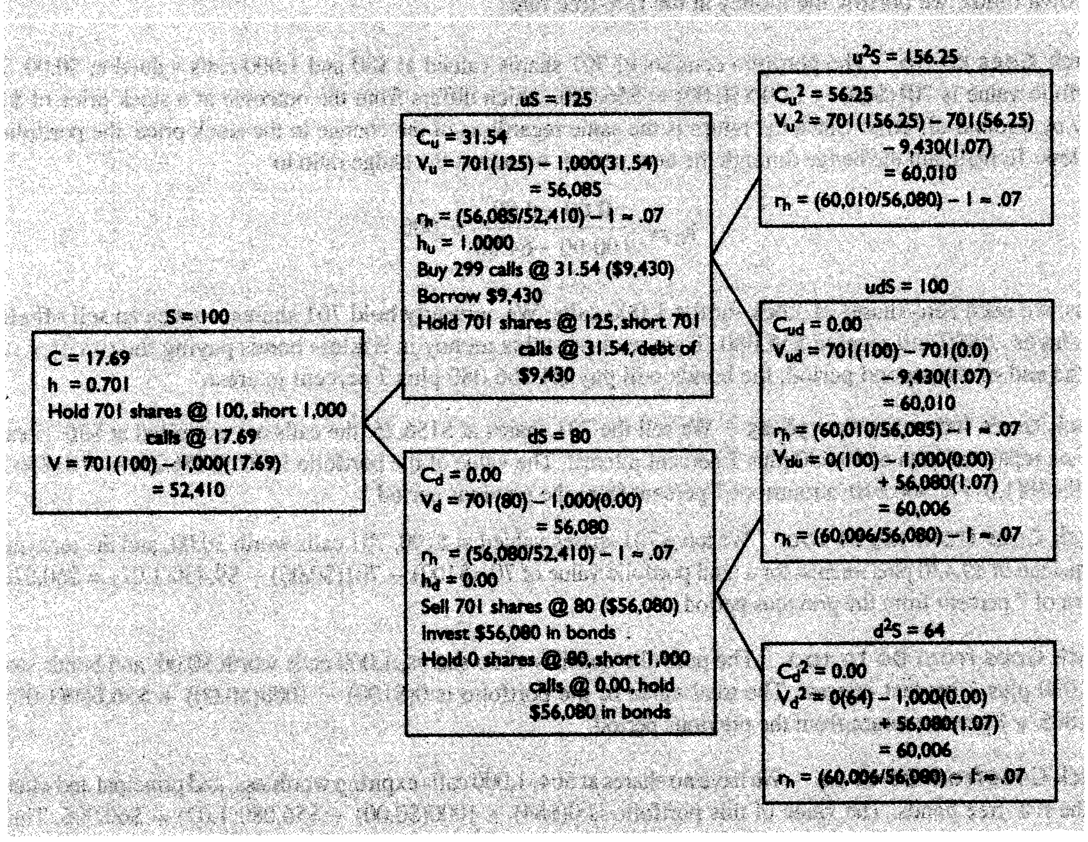
$$C = \frac{(0.6)31.54 + (0.4)0}{1.07} = 17.69$$

Note that the same call analyzed in the one-period world is worth more in the two-period world. Why? Recall from Chapter 3 that a call option with a longer maturity is never worth less than one with a shorter maturity and usually is worth more. If this principle did not hold here, something would have been wrong with the model.

Hedge Portfolio

Now consider a hedge portfolio. Figure 4.4 illustrates this process. It would be very helpful to keep an eye on the figure as we move through the example. Let the call be trading in the market at its theoretical fair value of $17.69. The hedge will consist of 1,000 short calls. The number of shares purchased at time 0 is given by the formula for h,

FIGURE 4.4 Two-Period Binomial Example



$$h = \frac{31.54 - 0.00}{125 - 80} = 0.701$$

Thus, we buy 701 shares of stock and write 1,000 calls. The transaction can be summarized as follows:

Buy	701 shares at \$100	= \$70,100 (assets);
write	1,000 calls at \$17.69	= –\$17,690 (liabilities);
	net investment	= \$52,410 (net worth).

Stock Goes to 125 The portfolio consists of 701 shares at \$125 and 1,000 calls at \$31.54. The value of the portfolio is 701(\$125) – 1,000(\$31.54) = \$56,085. Our investment has grown from \$52,410 to \$56,085. You should be able to verify that this is a 7 percent return, the risk-free rate. To maintain a hedge through the next period, we need to revise the hedge ratio. The new hedge ratio, h_u, is

$$h_u = \frac{56.25 - 0.00}{156.25 - 100} = 1 .$$

The new hedge ratio will be one share of stock for each call.

To establish the new hedge ratio, we need either 701 calls or 1,000 shares of stock. We can either buy back 299 calls, leaving us with 701, or buy 299 shares, giving us 1,000 shares. Since it is less expensive to buy the calls, let us buy back 299 calls at $31.54 each for a total cost of $9,430. To avoid putting out more of our own funds, we borrow the money at the risk-free rate.

Stock Goes to 80 The portfolio consists of 701 shares valued at $80 and 1,000 calls valued at $0.00. The portfolio value is 701($80) − 1,000($0.00) = $56,080, which differs from the outcome at a stock price of $125 only by a round-off error. Since the return is the same regardless of the change in the stock price, the portfolio is riskless. To maintain the hedge through the next period, we adjust the hedge ratio to

$$h_d = \frac{0.00 - 0.00}{100.00 - 64.00} = .000$$

Thus, we need zero shares of stock for the 1,000 calls. We currently hold 701 shares, so we can sell off all of the shares at $80 and receive $56,080. Then we invest this money in riskless bonds paying the risk-free rate. At the end of the second period, the bonds will pay off $56,080 plus 7 percent interest.

Stock Goes from 125 to 156.25 We sell the 701 shares at $156.25, the calls are exercised at $56.25 each, and we repay the loan of $9,430 plus 7 percent interest. The value of the portfolio is 701($156.25) − 701($56.25) − $9,430(1.07) = $60,010, a return of 7 percent from the previous period.

Stock Goes from 125 to 100 We have 701 shares valued at $100, 701 calls worth $0.00, and the repayment of the loan of $9,430 plus interest for a total portfolio value of 701($100) − 701($0.00) − $9,430(1.07) = $60,010, a return of 7 percent from the previous period.

Stock Goes from 80 to 100 The portfolio consists of no shares, 1,000 calls worth $0.00, and bonds worth $56,080 plus 7 percent interest. The total value of the portfolio is 0($100) − 1000($0.00) + $56,080(1.07) = $60,006, a 7 percent return from the previous period.

Stock Goes from 80 to 64 We have no shares at $64, 1,000 calls expiring worthless, and principal and interest on the risk-free bonds. The value of this portfolio is 0($64) + 1000($0.00) + $56,080(1.07) = $60,006. This is essentially the same amount received as in the other cases; the difference is due only to a round-off error. Thus, regardless of which path the stock takes, the hedge will produce an increase in wealth of 7 percent in each period.

Now let us consider what happens if the call is mispriced.

Mispriced Call in the Two-Period World

If the call is mispriced at time 0, the law of one price is violated and an arbitrage opportunity exists. If the call is underpriced, we should purchase it and sell short h shares of stock. If the call is overpriced, we should write it and purchase h shares of stock. Whether we earn the arbitrage return over the first period, the second period, or both periods, however, will depend on whether the call price adjusts to its theoretical fair value at the end of the first period. If it does not, we may not earn a return in excess of the risk-free rate over the first period. The call must be correctly priced at the end of the second period, however, because it expires at that time. It is completely out of the question that an option could be mispriced at expiration.

The two-period return will be the geometric average of the two one-period returns; that is, if 6 percent is the first-period return and 9 percent is the second-period return, the two-period return will be $\sqrt{(1.06)(1.09)} - 1 =$ 0.0749 or 7.49 percent. If one of the two returns equals the risk-free rate and the other exceeds it, the two-period return will exceed the risk-free rate. If one of the two returns is less than the risk-free rate and the other is greater, the overall return can still exceed the risk-free rate. The return earned over the full two periods will exceed the risk-free rate if the option is mispriced at time 0, the proper long or short position is taken, and the correct hedge ratio is maintained.

There are many possible outcomes of such a hedge, and it would take an entire chapter to illustrate them. We therefore shall discuss the possibilities only in general terms. In each case, we will assume that the proper hedge ratio is maintained and the investor buys calls only when they are underpriced or correctly priced and sells calls only when they are overpriced or correctly priced.

Suppose that the call originally was overpriced and is still overpriced at the end of the first period. Because the call has not fallen sufficiently to be correctly priced, the return over the first period actually can be less than the risk-free rate. Because the call must be correctly priced at the end of the second period, however, the return earned over the second period will more than make up for it. Overall, the return earned over the two periods will exceed the risk-free rate.

If the call originally is overpriced and becomes correctly priced at the end of the first period, the return earned over that period must exceed the risk-free rate. The return earned over the second period will equal the risk-free rate, because the call was correctly priced at the beginning of the second period and is correctly priced at the end of it. Thus, the full two-period return will exceed the risk-free rate.

If the call is overpriced at the start and becomes underpriced at the end of the first period, the return earned over that period will exceed the risk-free rate. This is because the call will have fallen in price more than is justified and now is worth considerably less than it should be. At this point, we should close out the hedge and initiate a new hedge for the second period, consisting of a long position in the underpriced call and a short position in the stock. We can invest the excess proceeds in risk-free bonds. The second-period return will far exceed the risk-free rate. The overall two-period return obviously will be above the risk-free rate.

Table 4.1 summarizes these results. Similar conclusions apply for an underpriced call, although the interpretation differs somewhat because the hedge portfolio is short.

Table 4.1 Hedge Results in the Two-Period Binomial Model

	Return from Hedge Compared to Risk-Free Rate		
	Period 1	Period 2	Two-Period
Options Overpriced at Start of Period 1			
Status of option at start of period 2			
Overpriced	Indeterminate	Better	Better
Correctly priced	Better	Equal	Better
Underpriced	Better	Better	Better

EXTENSIONS OF THE BINOMIAL MODEL

Pricing Put Options

We can use the binomial model to price put options just as we can for call options. We use the same formulas, but instead of specifying the call payoffs at expiration, we use the put payoffs at expiration. To see the difference, look at Figure 4.3. Then we simply replace every C with a P; likewise, we substitute P for C in each formula.

Let us consider the same problem we have been working on, but we shall price the two-period European put that has an exercise price of 100. The values of the put at expiration are as follows:

$$P_{u^2} = \text{Max}(0,100 - 156.25) = 0.00$$
$$P_{ud} = \text{Max}(0,100 - 100) = 0.00$$
$$P_{u^2} = \text{Max}(0,100 - 64) = 36.$$

Note how we use the put intrinsic value formula at expiration, the greater of zero or X minus the stock price. Now we step back to time 1. Using the same formulas we used for calls,

$$P_u = \frac{(0.6)0 + (0.4)0}{1.07} = 0$$

$$P_d = \frac{(0.6)0 + (0.4)36}{1.07} = 13.46$$

Again, the option value is a weighted average of its two possible values in the next period, where the weights are p and 1 − p, discounted back one period at the risk-free rate. Now we find the time 0 value to be

$$P = \frac{(0.6)0 + (0.4)13.46}{1.07} = 5.03$$

Now let us work through a hedge example. The principles are essentially the same as for hedging with calls, but instead of selling calls to hedge a long position in stock, we are buying puts. Calls move directly with the stock price, so to hedge, we have to sell calls. Puts, however, move inversely with the stock price, so to hedge stock, we buy puts.

The formula for the hedge ratio is the same as that for calls: the option price the next period if the stock goes up minus the option price the next period if the stock goes down, divided by the stock price in the next period if the stock price goes up, minus the stock price in the next period if the stock goes down. Thus, at time 0 the hedge ratio is

$$h = \frac{0 - 13.46}{125 - 80} = -0.299.$$

Since puts move opposite to stock, we should buy puts, so just ignore the minus sign. So we need to buy 299 shares for a long position in 1,000 puts. This will cost

$$\begin{aligned} 299(\$100) &= \$29{,}900 \text{ (in shares)} \\ 1{,}000(\$5.03) &= \$5{,}030 \text{ (in puts)} \\ \text{Total} \quad & \quad \$34{,}930. \end{aligned}$$

Stock Goes to 125 We now have 299 shares worth \$125 and 1,000 puts worth \$0.00 for a total value of 299(\$125) + 1,000(\$0.00) = \$37,375. This is approximately 7 percent more than our initial investment of \$34,930. Now the new hedge ratio will be

$$h_u = \frac{0 - 0}{156.25 - 100} = 0$$

So, we need no shares for our 1,000 puts. We sell the 299 shares, collecting 299(\$125) = \$37,375 and invest this money in risk-free bonds that will earn 7 percent.

Stock Goes to 80 We now have 299 shares worth \$80 and 1,000 puts worth \$13.46 for a total value of 299(\$80) + 1,000(\$13.46) = \$37,380. This is the same as when the stock went to 125, except for a round-off difference. Again, the return is 7 percent on our initial investment of \$34,930. Now the hedge ratio will be

$$h_d = \frac{0 - 36}{100 - 64} = -1.0000.$$

Again, ignore the minus sign. So now we need 1,000 shares and 1,000 puts, or 299 shares and 299 puts. Let us buy 701 shares, which will give us 1,000 shares and 1,000 puts. This purchase of stock will cost money so we borrow the funds at 7 percent. Now we have 1,000 shares worth \$80, 1,000 puts worth \$13.46, and a loan of \$56,080.

Stock Goes from 125 to 156.25 We have only a bond worth \$37,375(1.07) = \$39,991, which is a return of 7 percent for the period.

Stock Goes from 125 to 100 Again, we have only a bond worth \$37,375(1.07) = \$39,991, which is a 7 percent return for the period.

Stock Goes from 80 to 100 Now we have 1,000 shares worth \$100, 1,000 puts that are worthless, and we owe \$56,080(1.07) = \$60,006 on our loan. Thus, our total value is 1,000(\$100) + 1,000(\$0.00) − \$60,006 = \$39,994. This is the same total as the two outcomes above, except for a small round-off error. Again, the return for the period is 7 percent.

Stock Goes from 80 to 64 Now we have 1,000 shares worth \$64, 1,000 puts worth \$36, and we owe \$56,080(1.07) = \$60,006 on our loan. Thus, our total value is 1,000(\$64) + 1,000(\$36) − \$60,006 = \$39,994 the same total and 7 percent return as above.

So we see that as long as the hedge is maintained properly, we earn the risk-free return over each period. If the put were mispriced, we would take the appropriate position and hedge it with the stock. For example, if the put were selling for less than the model price, we would consider it underpriced. We would then buy it and hedge by buying the proper number of shares of stock. If the put were overpriced, we would sell it, but to hedge, we would need to sell short the stock.

American Puts and Early Exercise

The two-period binomial model is an excellent opportunity to illustrate how American options can be exercised early. Let us use the same two-period put with an exercise price of 100, but make it an American put. This means that at any point in the life of the option, we can choose to exercise it early if it is best to do so. That means at any point in the binomial tree when the put is in-the-money, we need to see if it is worth more to exercise it early.

For example, go back to the values calculated for the European put at time 1. Recall that they were

$$P_u = \frac{(0.6)0 + (0.4)0}{1.07} = 0.00 \text{ when the stock is at 125,}$$

$$P_d = \frac{(0.6)0 + (0.4)36}{1.07} = 13.36\text{, when the stock is at 80.}$$

When the stock is at 125 the put is out-of-the-money, so we do not need to worry about exercising it. When the stock is at 80, however, the put is in-the-money, and we have the right to exercise it. In fact, the put is in-the-money by \$20[Max(0, 100 − 80)], which is far more than its unexercised value of \$13.46. So we exercise it, which means that we replace the calculated value P_d of 13.46 with 20. Thus, we now have $P_u = 0$ and $P_d = 20$. Then the value at time 0 is

$$P = \frac{(0.6)0 + (0.4)20}{1.07} = 7.48.$$

We would also have to consider the possibility of exercising it immediately, but since it is at-the-money, there is no reason to exercise it today. Note that its value is considerably more than its value as a European put of 5.03.

Forming a hedge portfolio with American options follows the same procedure as forming a hedge portfolio for European options, except that we use the American option values to compute the portfolio values and hedge ratios.

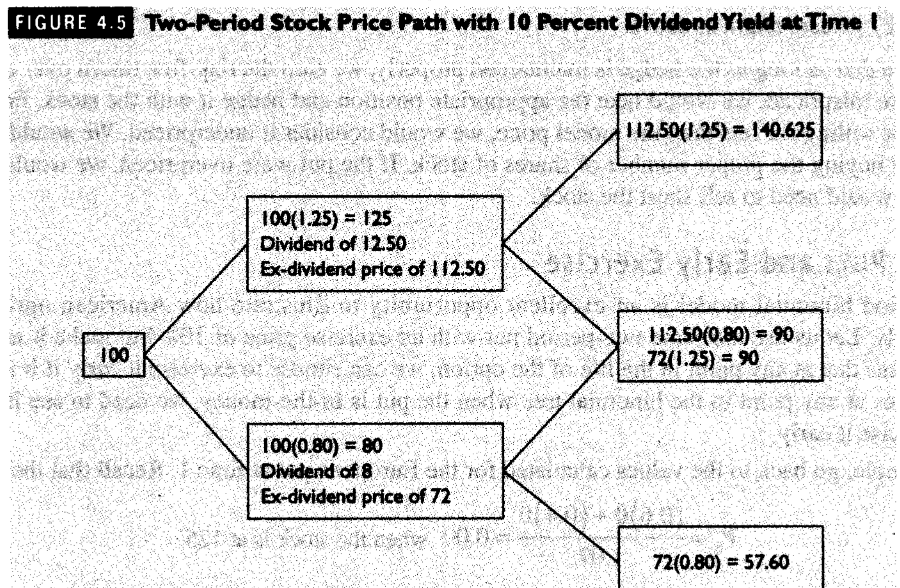
Dividends, European Calls, American Calls, and Early Exercise

For call options on stocks without dividends, there will, of course, never be an early exercise. Let us now consider how early exercise will affect American calls in the binomial model. To do so we must add a dividend.

There are a number of ways to incorporate dividends into the model. The simplest is to express the dividend as a yield of δ percent. Thus, when the stock price moves into the dividend period, it immediately declines by δ percent as it goes ex-dividend. We then use the ex-dividend stock prices in the binomial formulas. If the intrinsic value of the call before it goes ex-dividend exceeds the value of the call given by the binomial formula, the call should be exercised. Then the intrinsic value replaces the formula value.

Consider the same two-period problem we worked earlier in the chapter. Because we want to see a case where the call is exercised early, let us assume a fairly high dividend yield— say, 10 percent—and, let the dividend be paid and the stock go ex-dividend at time 1. At time 1, if the stock goes to 125, it then pays a 12.50 dividend and falls to 112.50. If the stock goes down to 80, it pays a dividend of 8.00 and falls to 72. The following period, its movement is based from values of either 112.50 or 72 and will be 140.625, 90, or 57.60. This process is shown in Figure 4.5.

FIGURE 4.5 Two-Period Stock Price Path with 10 Percent Dividend Yield at Time 1



The corresponding call prices at expiration are

$$C_{u^2} = \text{Max}(0, 140.625 - 100) = 40.625$$

$$C_{ud} = C_{du} = \text{Max}(0, 90.00 - 100) = 0$$

$$C_{d^2} = \text{Max}(0, 57.60 - 100) = 0$$

The European call prices after one period are

$$C_u = \frac{(0.6)\,40.625 + (0.4)0}{1.07} = 22.78$$

$$C_d = \frac{(0.6)\,0 + (0.4)0}{1.07} = 0.00$$

Thus, the European call value at time 0 is

$$C = \frac{(0.6)\,22.78 + (0.4)0}{1.07} = 12.77.$$

Note that this value is considerably less than its value without dividends of 17.69. Dividends will always reduce the value of a European option, because they represent a payout rather than a reinvestment of corporate cash flows for the purpose of generating growth in the stock price.

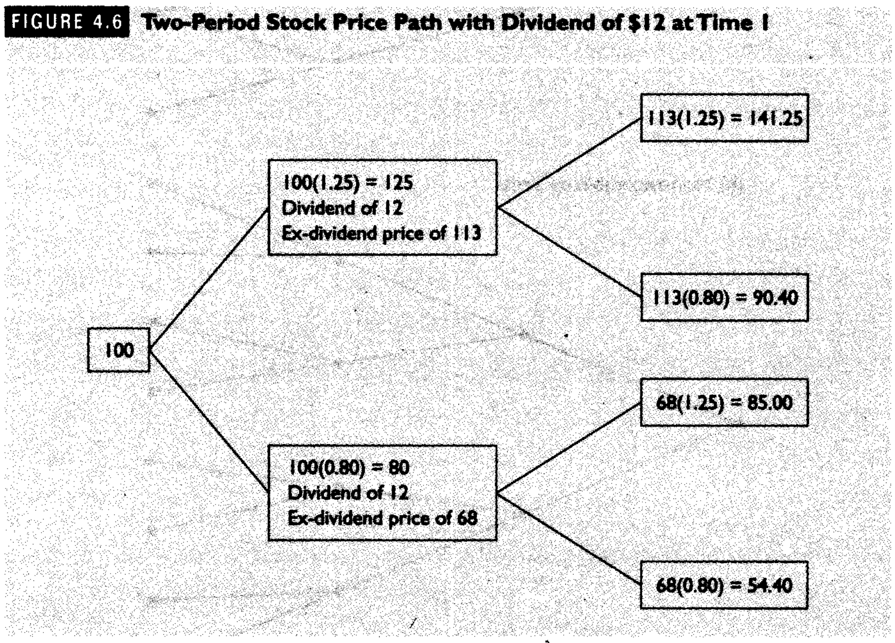
Now let the call be American. Let us move from time 0 to time 1 and let the stock move to 125. The firm is paying a dividend of 10 percent of 125, or 12.50, in just another instant. When that happens, the stock will fall to 112.50. Holding an American call, we have the right to exercise it before the stock goes ex-dividend, thereby paying 100 and receiving a stock worth 125. When the stock goes ex-dividend, its value drops to 112.50, but having acquired the stock before it goes ex-dividend, we are entitled to the dividend and have a net value of 25. Consequently, the option value at that point is 25. In other words, we simply choose to exercise the option before it goes ex-dividend and claim its intrinsic value of 25. Consequently, we, replace the binomial formula value of 22.78, previously computed in the European option example, with 25. Thus, we now have $C_u = 25$. In the time 1 outcome where the stock falls to 80, we cannot justify early exercise as the call is out-of-the-money.

Stepping back to time 0, we find that the value of the American call is, therefore[4].

$$C = \frac{(0.6)\,25 + (0.4)0}{1.07} = 14.02.$$

As an alternative approach, suppose we simply have the firm pay a specific dollar dividend at time 1. Let us make it a dividend of 12. Now we run into a slight problem. As Figure 4.6 shows, at time 1 the stock goes ex-dividend to a value of either 113 or 68. If the stock is at 113 and goes down at time 2, its new price will be 113(0.80) = 90.40. If the stock is at 68 and goes up at time 2, its new price will be 68(1.25) = 85. Consequently, the middle state at time 2 will not be the same regardless of where the stock was at time 1. In the example illustrated here, this is not really a problem: The option value can still be computed in the normal manner. When we expand the model to include a large number of periods, however, this problem will greatly increase the computational requirements.

FIGURE 4.6 Two-Period Stock Price Path with Dividend of $12 at Time 1



[4]You may recall that the value of 14.02 was also the value of the one-period call. There is no connection between the one-period European option value and the two-period American option value other than that, in the latter case, we exercised the option at time 1, thereby effectively making it a one-period option.

A binomial tree in which an up move followed by a down move puts you in the same location as a down move followed by an up move is called a recombining tree. When an up move followed by a down move does not put you in the same location as a down move followed by an up move, the tree is called non-recombining tree. For a tree with n time periods, a recombining tree will have n + 1 final stock prices. There will be 2^n distinct paths taken to reach the final stock prices, but some of the paths will leave you in the same location; consequently, we would not have to work our way through each path to identify the final outcome. This greatly simplifies and reduces the computations in a tree with a large number of time periods. A non-recombining tree will have 2^n distinct paths and, therefore, 2^n final outcomes. If n is large, this number becomes astronomical quite quickly and can pose severe computational difficulties, even for the fastest computers. Figure 4.7 illustrates a three-period recombining and non-recombining tree.

One special trick can greatly simplify the binomial computations for the case of an American option with dividends. Recall that in Chapter 3, in cases with dividends, we subtracted the present value of the dividends from the stock price and used this adjusted value in our formulas. We can do the same here. We are simply assuming that the dividends are fully predictable and that it is the stock price minus the present value of the dividends that follows the binomial process.

FIGURE 4.7 Recombining and Non-Recombining Three-Period Binomial Trees

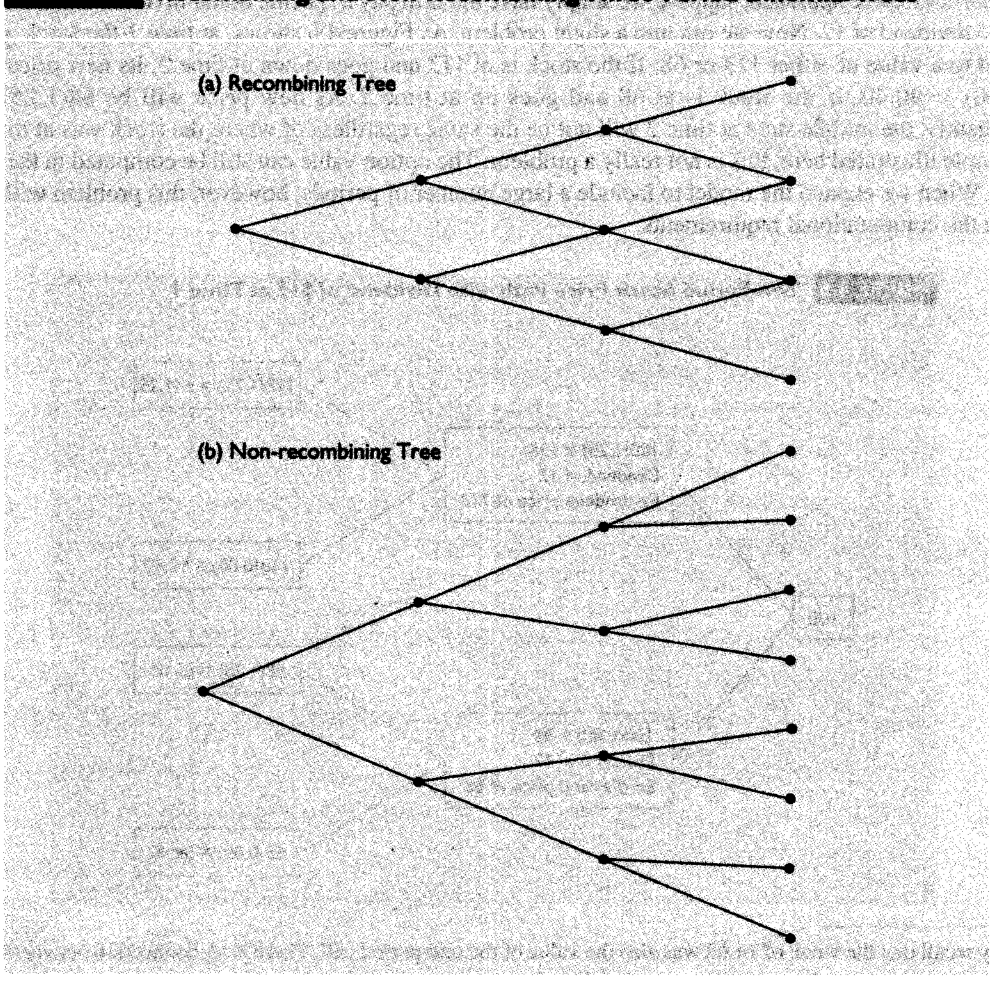


FIGURE 4.8 Two-Period Stock Price Path with Dividend of $12 at Time 1 and Stock Price Minus Present Value of Dividends Follows the Binomial Process

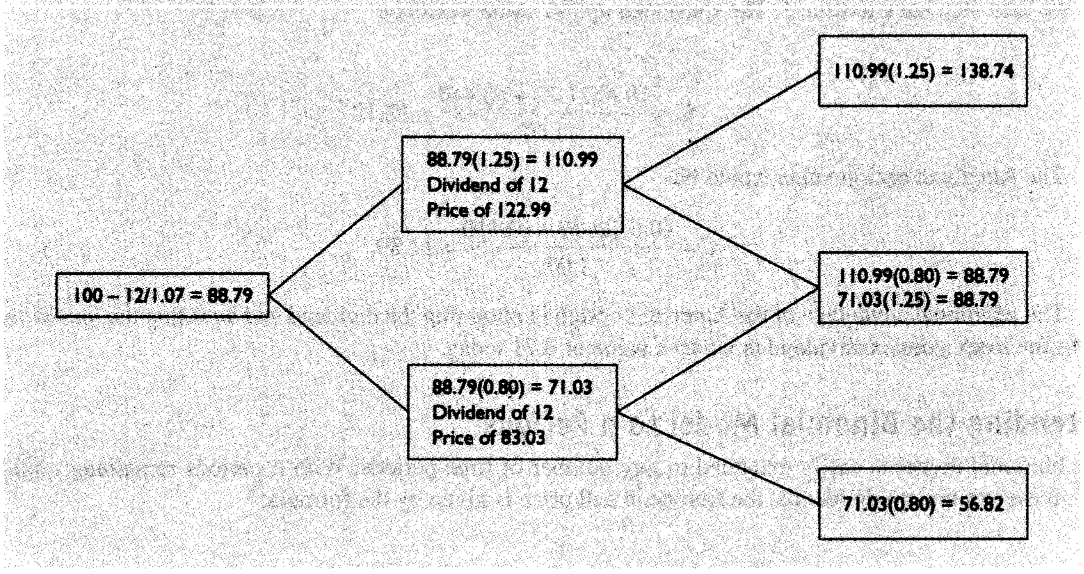


Figure 4.8 illustrates this approach. The current stock price is \$100, but we adjust that value to 100 − 12/1.07 = 88.79. Then the value 88.79 goes up or down by the factor 1.25 or 0.80. Thus, at time 0, the stock is observed to be 100, which consists of a present value of the dividend of 12/1.07 = 11.21 and the remainder of the price, 88.79, which reflects the stock's growth potential. At time 1, if the stock goes up, the adjusted stock price is 88.79(1.25) = 110.99. Before the stock goes ex-dividend, its actual price is 110.99 + 12 = 122.99. It then falls immediately to 110.99 as it goes ex-dividend. If at time 1, the stock went down, the adjusted price will be 88.79(0.80) = 71.03. Before it goes ex-dividend, it will be worth 83.03. Then it goes ex-dividend and will fall to 71.03. We then see in the figure that at time 2, the middle point will be the same, with the stock at 88.79 regardless of which path was taken to get there.

Now we can easily calculate the call value:

$$C_{u^2} = \text{Max}\,(0,\ 138.74 - 100) = 38.74$$

$$C_{ud} = \text{Max}\,(0,\ 88.79 - 100) = 0.00$$

$$C_{d^2} = \text{Max}\,(0,\ 56.82 - 100) = 0.00$$

Now we step back to time 1. If the call is European, we calculate its values to be

$$C_u = \frac{(0.6)\,38.74 + (0.4)0}{1.07} = 21.72.$$

$$C_u = \frac{(0.6)0 + (0.4)0}{1.07} = 0.$$

If the call is American, however, at time 1 when the stock is at 122.99, we have the opportunity to exercise the call just before the stock goes ex-dividend. In that case we obtain a payoff of 22.99, resulting from acquisition of the stock for $100, collection of the dividend of $12, and the stock falling to a value of $110.99. Since the payoff from exercising of 22.99 is greater than the value if not exercised, we use 22.99 for C_u. We then step back to time 1. The European option value would be

$$C = \frac{(0.6)21.72 + (0.4)0}{1.07} = 12.18.$$

The American option value would be

$$C = \frac{(0.6)22.99 + (0.4)0}{1.07} = 12.89.$$

The additional advantage of the American option in obtaining the dividend and avoiding the loss in value when the stock goes ex-dividend is worth a value of 0.71 today.

Extending the Binomial Model to n Periods

The binomial model is easily extended to any number of time periods. With n periods remaining until the option expires and no dividends, the European call price is given by the formula:[5]

$$C = \frac{\sum_{j=0}^{n} \frac{n!}{j!(n-j)!} p^j (1-p)^{n-j} \operatorname{Max}\left[0, u^j d^{n-j} S - X\right]}{(1+r)^n}$$

This seemingly difficult formula actually is not nearly as complex as it appears. It simply captures all of the possible stock price paths over the n periods until the option expires. Consider the example from the text in a three-period world where j will go from 0 to 3. First, we find the summation of the following terms.

For $j = 0$,

$$\frac{3!}{0!3!}(.6)^0(.4)^3 \operatorname{Max}\left[0, (1.25)^0(.80)^3 100 - 100\right] = 0.$$

For $j = 1$,

$$\frac{3!}{1!2!}(.6)^1(.4)^2 \operatorname{Max}\left[0, (1.25)^1(.80)^2 100 - 100\right] = 0.$$

For $j = 2$,

$$\frac{3!}{2!1!}(.6)^2(.4)^1 \operatorname{Max}\left[0, (1.25)^2(.80)^1 100 - 100\right] = 10.80.$$

For $j = 3$,

$$\frac{3!}{3!0!}(.6)^3(.4)^0 \operatorname{Max}\left[0, (1.25)^3(.80)^0 100 - 100\right] = 20.59.$$

[5]The meaning of factorial or n! is $n! = n(n-1)(n-2)\ldots 3(2)(1)$.